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THESIS

REAL TIME MARINE GAS TURBINE
SIMULATION FOR ADVANCED CONTROLLER
DESIGN

by

Stephen D. Metz

September 1989

Thesis Advisor: David L. Smith

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A modeling method is shown which utilizes real time sequential linearizations to approximate the true nonlinear response of the NPS Boeing 502-6A test facility. A validation of this simulation approach is presented. The method has immediate application to advanced controller design, especially to the design of modern regulators (Linear Quadratic), model reference controllers, and real time diagnostics.

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Real Time Marine Gas Turbine Simulation
for Advanced Controller Design

by

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requirements for the degree of

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ABSTRACT

The Marine Gas Turbine control systems in present use in the U. S. Navy are of such significant technological age that new design techniques could lead to more optimum performance and increased plant efficiency. To this end, a new real time Marine Gas Turbine simulation method is needed for advanced controller design and implementation.

A modeling method is shown which utilizes real time sequential linearizations to approximate the true nonlinear response of the NPS Boeing 502-6A test facility. A validation of this simulation approach is presented. The method has immediate application to advanced controller design, especially to the design of modern regulators (Linear Quadratic), model reference controllers, and real time diagnostics.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS AND ABBREVIATIONS

E - Fuel Combustion Energy Realized at HP Turbine

JD - Dynamometer Torque

JG - Gas Generator Torque

Ma - Mass Flowrate of Air

Maf - Mass Flowrate of Air/Fuel Mixture

Mf - Mass Flowrate of Fuel

Ng - Gas Generator Speed

Ns - Power Turbine/Dynamometer Speed

Qc - Compressor Torque

Qd - Dynamometer Torque

Qfpt - Free Power Turbine Torque

Qhpt - High Pressure Turbine Torque

P2 - Compressor Discharge Pressure

P4 - High Pressure Turbine Discharge Pressure

T2 - Compressor Discharge Temperature

T4 _ High Pressure Turbine Discharge Temperature

Note: 1. Lower case variables represent normalized variables.

2. __B denotes normalizing values. (i.e. JGB, NGB, QCB, etc.)

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I. INTRODUCTION

Conventional control theory is limited to time invariant single-input-single-output systems and is usually based on a frequency domain design approach. Modern control theory, on the other hand, can be well applied to multiple-input-multiple-output, non-linear gas turbine systems which are now in use and it relies on a much more applicable time domain approach.[1] The feasibility of modern control theory such as Linear Quadratic Regulator (LQR) theory, in the application of marine gas turbines for propulsion has been demonstrated by past work at the Naval Postgraduate School in Monterey, California.[2-4] For this application to be successful a simple, accurate, and robust real time marine gas turbine simulation must be developed.

This thesis introduces a modeling approach to be applied in developing a real time simulation of engine response to meet the requirements of simplicity, accuracy, and robustness. It is simple in that it is only first order dependent, accurate in that it provides acceptable (+/- 10%) response for use in application of advanced engine control design, and robust in that it works well over the entire operational range of the gas turbine system.

II. PAST WORK IN GAS TURBINE MODELING

Past work in Gas Turbine Modeling is discussed in terms of two areas: first, a brief discussion on gas turbine modeling in general is presented. This is followed by a discussion on the previous work completed at the Naval Postgraduate School in Monterey, California.

Since most marine gas turbines have been derived from aviation sources, most of the early gas turbine modeling work was accomplished in the aviation arena. In the 1970's the Naval Gas Turbine Ship Propulsion Dynamics and Control Systems Research and Development Program was started to address the issues of designing Marine Gas Turbines for future use in the Navy.[5] This included the dynamic modeling of the response of a marine gas turbine. Prior to this program, most of the work had been steady state modeling. However, the Navy realized that dynamic modeling was needed for advanced controller design. The program was successful in developing a gas turbine machinery dynamics and control system data base. From this work a computer-based propulsion control testbed was developed for use in gas turbine controller development. Cut and try linear control design then ensued, which resulted in a gain adjusted controller design in use in today's fleet. This approach resulted in a linear Proportional-Integral theory being applied to a nonlinear system. The results were deemed adequate for the Navy's uses.

A new approach to this modeling is to develop a method to accurately model the propulsion plant nonlinear responses with a real time simulator. This would help eliminate or alleviate problems associated with the varying propeller loads felt in the gas turbine by predicting the responses before they actually occur, thus allowing more acurate control. More accurate control would mean better engine response to these loads and would, in turn, mean less wear, thus leading to longer engine life as well as better engine fuel efficiency. This is the direction taken at the Naval Postgraduate School, (NPS), in the gas turbine modeling problem.

The previous work started with Phillip Johnson in 1985 in which he conducted the development and implementation of a load control system (e.g. a water brake dynamometer) to emulate the shaft and propeller of a marine gas turbine.[6] The next step was taken by James Roger also in 1985. He used a simple multiport diagram and the Continous System Modeling Program (CSMP III) to incorporate an improved model for the dynamometer unloading valve into the existing system dynamic model.[7] These two works formed the base for the next step performed by Vincent Herda in 1986. Herda developed a better multiport model and used it to develop a linear state space model and a nonlinear dynamic propulsion model.[2] Robert Miller followed in the same year using Herda's models and examined the feasibility of using Linear Quadratic Regulator (LQR) control design theory for a small gas turbine, specifically the Boeing 502-6A test bed installation at NPS.[3] Finally Vincent Stammetti in 1988 developed the first real time dynamic simulator for the gas turbine and compared an LQR controller to a classical Proportional Integral (PI) regulator based on his results with the simulator.[4] To summarize, Herda validated cause and effect through the use of his new multiport model, shown in figure 1 , Miller validated the steady-state model method and Stammetti followed with the first linear model structure. It is important to note that these models were only accurate for one run of simulation and were not applicable over the whole range of operation of the gas turbine. Their main point however, was to validate and show the feasibility of developing a better model for use in modern controller design to update those in current use today. To this end they were successful. This paper will continue this work and attempt to develop a simple, robust, and accurate real time marine gas

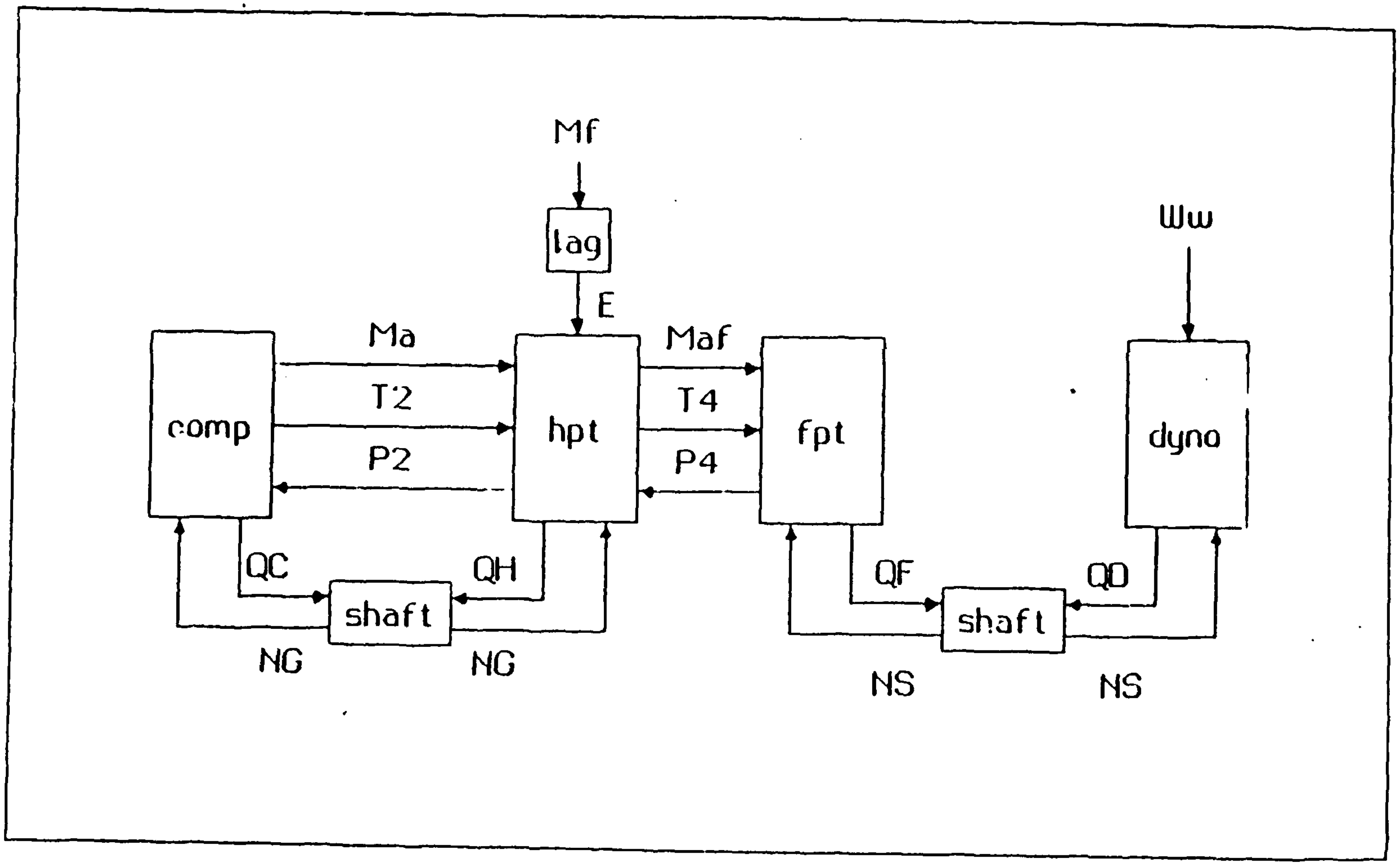


Figure 1. Herda's Multiport Diagram.

turbine simulator to be used for advanced controller design.

III. MODEL STRUCTURE AND COARSE TUNING

A. Modeling

The gas turbine plant to be modeled was the 175 horsepower Boeing 502-6A which was assembled in a test bed configuration at the Naval Postgraduate School in Monterey, California. It was coupled to a Clayton 17-300 water brake dynamometer which was used to simulate the shaft and propeller loading on a marine gas turbine. The gas turbine/water brake assembly is shown in figure 2. The gas turbine itself was composed of two aerodynamically coupled sections, a gas generating section and a power output section. The gas generator had three major components. These were the compressor, the burner, and the high pressure turbine shown in figure 3. Connected by gearing off the gas generator shaft was the accessory section consisting of the fuel pump, lube oil pump, governor, tachometer generator, and the starter. The compressor was a single stage centrifugal compressor and was coupled to the high pressure turbine aerodymanically via two through-flow type, cross-connected combustion chambers. The high pressure turbine was a single stage axial flow high pressure turbine. It was aerodynamically connected to the low pressure or free power turbine which was also an axial flow turbine. The free power turbine output shaft was mechanically coupled to the water brake dynamometer which absorbs the energy of

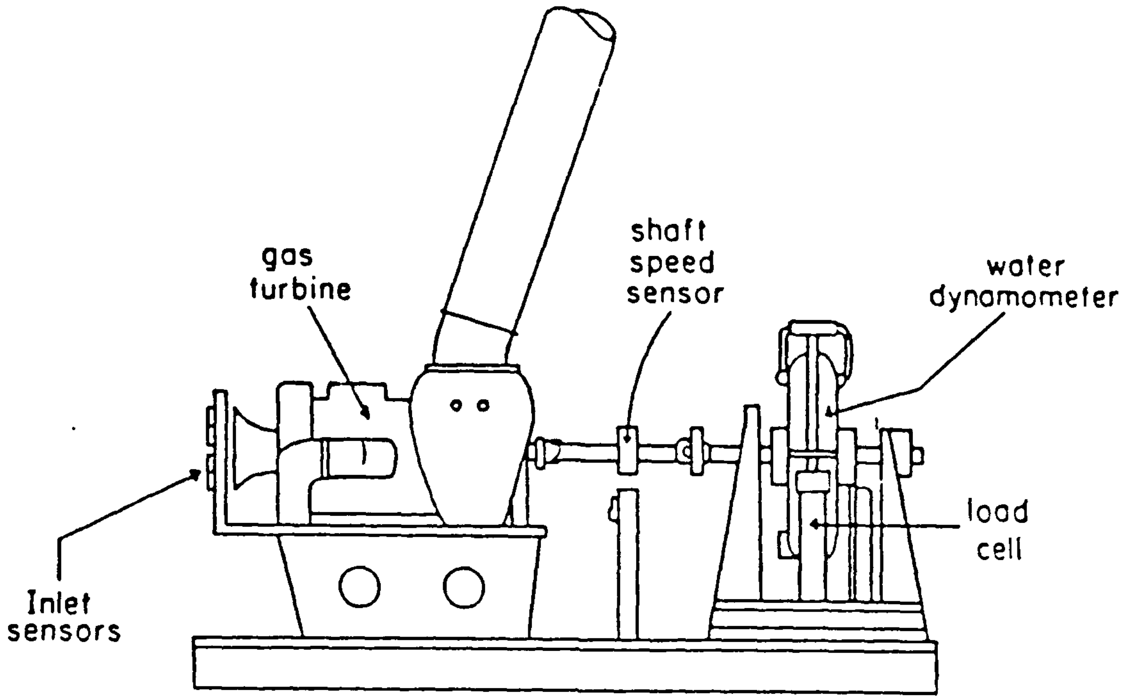


Figure 2. *Test Bed Configuration.*

175 Hp Boeing 502-6A gas turbine

Single stage centrifugal compressor

Two single stage axial turbines

Clayton 17-300 water brake dynamometer

Assorted sensors

Pressure

Temperature

Output torque

Rotational speeds

Fuel flow rate

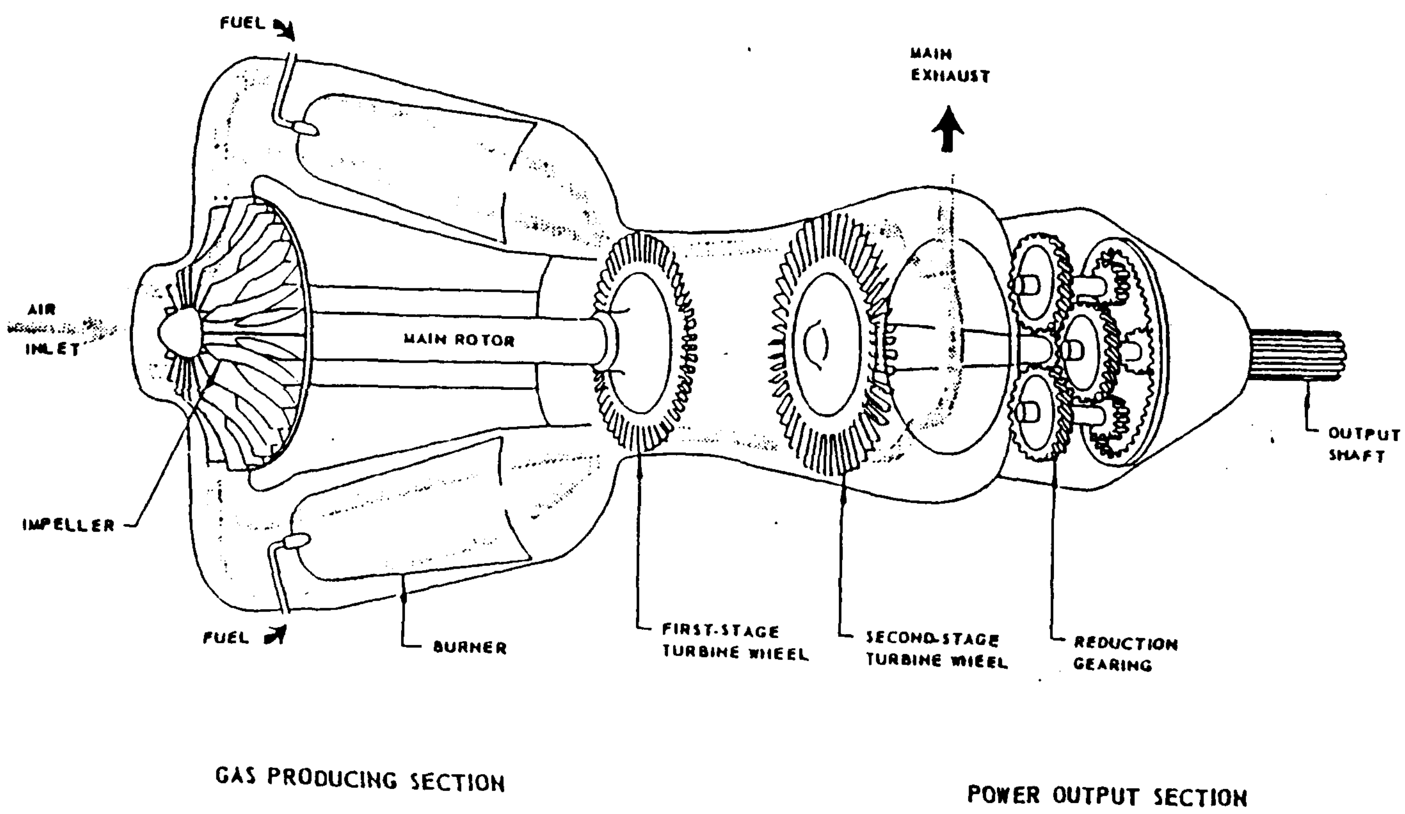


Figure 3. Gas Turbine Components.

the free power turbine in simulating the action of a drive shaft and propeller.

The modeling began with Herda's cause and effect multiport diagram shown in figure 1. The importance of this diagram to the method can hardly be overstated. Each of the components, the compressor, high pressure and free power turbines, and the gas generator and dynamometer shafts, were modeled by describing their outputs in terms of their inputs in equation form. For example, the compressor inputs are gas generator speed (Ng) and the high pressure turbine inlet pressure (P2) and the outputs are mass flow rate of air (Ma), high pressure turbine inlet temperature (T2), and compressor torque (Qc). The compressor block diagram is shown in figure 4.

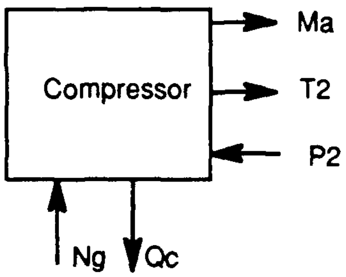


Figure 4. Compressor Block Diagram.

It is important to note that these input/output quantities as well as those for the other components must be measurable or calculable. They were recorded and or calculated over the operating range of the gas turbine (shown in figure 5), and the results are tabulated in table 1. For clarity, the operating region of the turbine is shown in figure 5.

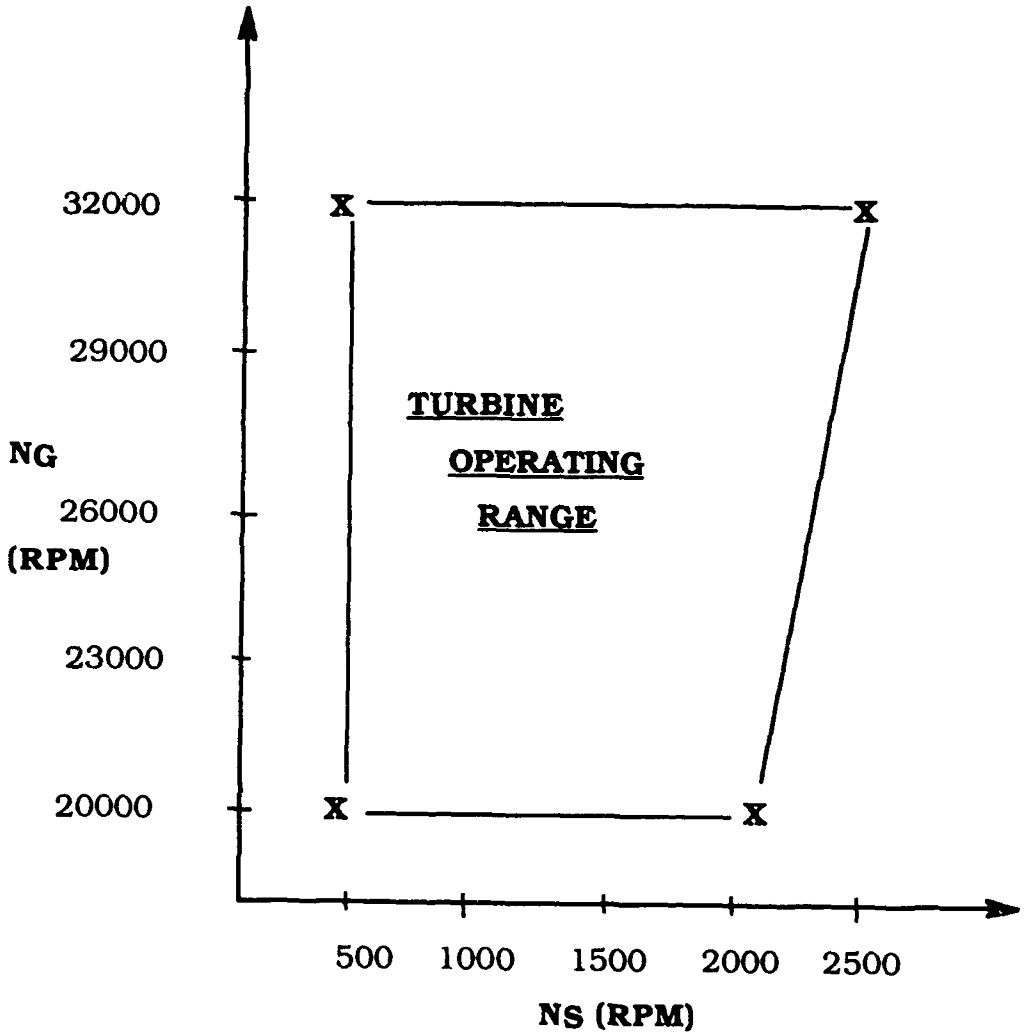


Figure 5. Gas Turbine Operating Range.

Ng rpm	Ns rpm	Qd/ Qfpt ftlbf	T2 f	P2 psig	T4 f	P4 psig	Mf lbm/hr	Ma lbm/s	Qc/ Qhpt ftlbf
20040	500	125	146.73	6.63	868	.85	76.98	1.73	60.564
20020	1003	95	146.68	6.68	882	.94	78.39	1.71	60.981
20000	1504	70	146.90	6.70	880	.96	79.40	1.71	62.764
19930	2002	40	147.15	6.75	876	.95	79.40	1.71	63.326
20000	2267	20	146.93	6.60	878	.90	79.40	1.71	64.020
23050	511	180	177.45	9.18	877	1.15	91.45	2.05	65.696
23060	1009	145	177.20	9.25	902	1.30	94.49	2.03	67.455
23060	1500	115	176.15	9.40	902	1.35	94.98	2.03	69.329
23160	2008	80	175.30	9.35	895	1.35	94.98	2.03	70.439
23020	2509	40	174.98	9.33	897	1.30	94.98	2.01	72.434
26010	506	245	205.90	12.20	886	1.59	108.31	2.42	72.828
26090	1006	200	206.58	12.55	922	1.72	110.83	2.38	71.378
26080	1502	170	208.03	12.43	917	1.80	113.60	2.35	74.238
26090	2000	130	207.40	12.55	921	1.90	113.60	2.35	76.306
26090	2510	100	208.50	12.55	926	1.85	113.60	2.36	75.667
29020	509	320	244.18	16.05	913	2.10	128.67	2.74	85.802
29060	1010	280	247.33	16.05	960	2.20	131.44	2.69	80.095
29010	1504	230	245.85	16.35	967	2.40	135.47	2.67	78.711
29050	2000	195	244.36	16.45	970	2.50	136.47	2.66	81.167
29060	2515	160	242.40	16.40	956	2.50	137.24	2.66	85.736
32030	509	420	280.65	20.50	978	2.70	157.84	3.06	99.810
32020	1010	365	283.03	20.80	1002	2.85	161.86	3.03	94.685
32020	1510	310	281.05	20.90	1039	3.00	165.11	3.01	90.221
32010	2017	275	283.50	20.95	1024	3.30	167.60	3.04	91.634
32000	2510	240	284.70	20.85	1044	3.30	168.34	2.99	89.646

(A)

ng	ns	qd/qfpt	t2	p2	t4	p4	mf	ma	qc/qhpt	maf
0.77077	0.33333	0.56818	0.68247	0.47357	0.90795	0.44737	0.62841	0.73305	0.75705	0.72371
0.77000	0.66867	0.43182	0.68223	0.47714	0.92259	0.49474	0.63992	0.72458	0.76226	0.71561
0.76923	1.00267	0.31818	0.68326	0.47857	0.92050	0.50526	0.64816	0.72458	0.78455	0.71573
0.76654	1.33467	0.18182	0.68442	0.48214	0.91632	0.50000	0.64816	0.72458	0.79157	0.71573
0.76923	1.51133	0.09091	0.68340	0.47143	0.91841	0.47368	0.64816	0.72458	0.80025	0.71573
0.88654	0.34067	0.81818	0.82535	0.65571	0.91736	0.60526	0.74653	0.86864	0.82120	0.85760
0.88692	0.67267	0.65909	0.82419	0.66071	0.94351	0.68421	0.77135	0.86017	0.84319	0.84969
0.88692	1.00000	0.52273	0.81930	0.67143	0.94351	0.71053	0.77535	0.86017	0.86661	0.84975
0.89077	1.33867	0.36364	0.81535	0.66786	0.93619	0.71053	0.77535	0.86017	0.88049	0.84975
0.88538	1.67267	0.18182	0.81386	0.66643	0.93828	0.68421	0.77535	0.85169	0.90542	0.84148
1.00038	0.33733	1.11364	0.95767	0.87143	0.92678	0.83684	0.88416	1.02542	0.91035	1.01243
1.00346	0.67067	0.90909	0.96084	0.89643	0.96444	0.90526	0.90473	1.00847	0.89222	0.99619
1.00308	1.00133	0.77273	0.96758	0.88786	0.95920	0.94737	0.92735	0.99576	0.92797	0.98411
1.00346	1.33333	0.59091	0.96465	0.89643	0.96339	1.00000	0.92735	0.99576	0.95382	0.98411
1.00346	1.67333	0.45455	0.96977	0.89643	0.96862	0.97368	0.92735	1.00000	0.94584	0.98825
1.11615	0.33933	1.45455	1.13572	1.14643	0.95502	1.10526	1.05037	1.16102	1.07252	1.14700
1.11769	0.67333	1.27273	1.15037	1.14643	1.00418	1.15789	1.07298	1.13983	1.00119	1.12666
1.11577	1.00267	1.04545	1.14349	1.16786	1.01151	1.26316	1.10588	1.13136	0.98389	1.11886
1.11731	1.33333	0.88636	1.13656	1.17500	1.01464	1.31579	1.11404	1.12712	1.01459	1.11484
1.11769	1.67667	0.72727	1.12744	1.17143	1.00000	1.31579	1.12033	1.12712	1.07170	1.11493
1.23192	0.33933	1.90909	1.30535	1.46428	1.02301	1.42105	1.28849	1.29661	1.24762	1.28258
1.23154	0.67333	1.65909	1.31642	1.48571	1.04812	1.50000	1.32131	1.28390	1.18356	1.27065
1.23154	1.00667	1.40909	1.30721	1.49286	1.08682	1.57895	1.34784	1.27542	1.12776	1.26275
1.23115	1.34467	1.25000	1.31860	1.49643	1.07113	1.73684	1.36816	1.28814	1.14542	1.27544
1.23077	1.67333	1.09091	1.32419	1.48928	1.09205	1.73684	1.37420	1.26695	1.12057	1.25486

(B)

(A) Raw data
(B) Normalized data

Table 1. Steady State Data Taken Over Operating Range
(Ng 20k -32k rpm, Ns 500 - 2500 rpm)

At this point it should be noted that the data of table 1A was normalized according to each of the variable's midpoint for ease of understanding each variable's relative impact with respect to the others. Once normalized, the variables were are denoted using lower case letters. The resulting equation forms are:

$$Ma = f_1\ (Ng,\ P2), \qquad (1)$$

$$T2 = f_2\ (Ng,\ P2), \qquad \text{and} \qquad (2)$$

$$Qc = f_3\ (Ng,\ P2). \qquad (3)$$

The normalization values are shown in table 2.

Table 2. Normalization Points.

Ng = 26000	Ns = 1500
Qd/Qfpt = 220	T2 = 215
P2 = 14	T4 = 956
P4 = 1.9	Mf = 122.5
Ma = 2.36	Qc/Qhpt = 80

The next step was to distinguish between steady and dynamic components. Once this is complete, the component modeling can proceed. The rest of the component equation forms are shown in table 3.

Table 3. Component Equations.

Gas Generator Shaft:	$Ng = f_4\ (Qhpt, Qc)$	(4)
Dynamometer Shaft:	$Ns = f_5\ (Qfpt, Qd)$	(5)
Free Power Turbine:	$P4 = f_6\ (Ns, T4, Maf)$	(6)
	$Qfpt = f_7\ (Ns, T4, Maf)$	(7)
High Pressure Turbine:	$E = f_8\ (Mf)$	(8)
	$Maf = f_9\ (Ma, Mf)$	(9)
	$T4 = f_{10}\ (Ma, T2, P4, Ng, E)$	(10)
	$P2 = f_{11}\ (Ma, T2, P4, Ng, E)$	(11)
	$Qhpt = f_{12}\ (Ma, T2, P4, Ng, E)$	(12)

B. State Selections

State variables are the smallest set of variables which are required to ascertain the state or condition of a dynamic system.[1] The knowledge of these variables at an initial point in time (t_0), when combined with the input as time proceeds ($t > t_0$), will completly describe the systems behavior. The state of a system at any time (t) is indepentent of any prior state and input which precede the initial time point (t_0)[1]. Here, the turbine and load is the dynamic system and the states, when properly applied, will accurately describe the dynamic behavior of the system. Significant prior research has determined that only three states are necessary to sufficiently describe the operating condition of the NPS gas turbine.[8] In reality however, only two states are necessary (ng, and ns) since the third state (e) represents a short first order lag of the input fuel (mf). This time lag is due to the burner action delay between the input of additional fuel and the thermodynamic energy output felt in the turbine.

It is our goal to relate all of the dependent variables (p2, p4,t2,mf,...) to the three state variables ng, ns, and e to allow us to use the state space equation $x = Ax + Bu$ (ref. 9) to describe the transient behavior of our system. Here x is the state vector and is composed of state variables ng, ns, and e, u is the input vector and is composed of mf and qd. Refering back to figure 1, note that we have redrawn Herda's system boundary at qd in order to have a more measurable input. Again, from prior research, it was known that each of the states is associated with a significant time lag in the system response. In this way, we knew that the fuel input component, the rotor shaft, and the output shaft must be regarded as dynamic components. We also knew that the remaining components could be regarded as instantaneous (memoryless), or steady components. The steady and dynamic components are shown in table 4.

Table 4. Steady and Dynamic Components.

Steady Components:				
compressor	ma	t2	qc	
high pressure turbine	p4	qfpt		
power turbine	t4	p2	qhpt	maf
Dymamic Components:				
rotor shaft	ng			
load shaft	ns			
burner lag	e			

C. Steady Components

While the steady components of the model were given above as non-linear functions, in this section, we reduce these to linear expressions, where the coefficients were regressed and varried according to the engine operating point.

The steady component equation form selection was accomplished through the use of an equation fitting program (Minitab), and plotting of each of the input variables versus an output variable minus the residual. If the residual plot displayed a parabolic shape, it was an indication that the input variable function in question was of a higher order. In the case of the equations with only two input variables a different method was used to fit the data. That is, one at a time, each of the input variables were held constant and the other was plotted versus the output variable. Both methods are developed further in Appendix C. This resulted in accurate data fit equations.

To simplify the equation reduction methodology, the steady component equations were kept in the form:

$$x = c1\ y + c2\ z \tag{13}$$

For example, the compressor equations appeared as follows:

$$t2 = c1\ ng^2 + c2\ ng + c3\ p2 + c4 \tag{14}$$

$$ma = c5\ ng^2 + c6\ ng + c7\ p2 + c8 \quad \text{and} \tag{15}$$

$$qc = c9\ ng^2 + c10\ ng + c11\ p2 + c12 \tag{16}$$

where the coefficients c1 thru c12 were the values from the equation fits. It is important to note the strong dependency of the state ng. This will appear again in the decoupling equations developed at the end of this chapter. The next phase of development was the formulation of the dynamic component models.

D. Dynamic Components

The dynamic components are derived from the gas generator shaft, the dynamometer shaft, and the lag relationship between mf and e. The dynamic components were modeled with differential equations which, when integrated over time will determine the state of the system. The dynamic components took the form:

$$ng = qhpt-qc \quad (17)$$

$$ns = qfpt-qd \quad (18)$$

and $$e = (mf-e)/T. \quad (19)$$

These, along with the previous equations of the steady components, were next linearized to form a dynamic equation set for the simulation model.

E. Linearized Dynamic Equation Set

The linearized dynamic equation set was developed from the perturbational or derivative values of the steady state equations (symbolically, the perturbational variables will be proceeded by a "d"). The compressor equations are used below to demonstrate the formulation of the dynamic equations. Recall that the t2 steady state equation was:

$$t2 = c1\ ng^2 + c2\ ng + c3\ p2 + c4 \quad (14)$$

So, the dynamic equation took the form:

$$dt2 = [\ \partial t2/\partial ng\ ]\ dng + [\ \partial t2/\partial p2\ ]\ dp2 \quad (20)$$

Here the partial differentials will be denoted with upper case letters so that the resulting form of equation 20 is:

$$dt2 = A1\ dng + A2\ dp2. \quad (21)$$

The rest of the steady and dynamic equations previously developed were also converted to dynamic form in this manner. The results appear below in table 5.

Table 5. Dynamic Equation Set.

<u>Compressor</u>

$$dt2 = A1\ dng + A2\ dp2 \quad \text{............} \quad (22)$$

$$dma = B1\ dng + B2\ dp2 \quad \text{..........} \quad (23)$$

$$dqc = C1\ dng + C2\ dp2 \quad \text{...........} \quad (24)$$

<u>Gas Generator Shaft</u>

$$JGB\ dng = dqhpt\text{-}dqc \quad \text{......................} \quad (25)$$

<u>Dynamometer Shaft</u>

$$JDB\ dns = dqfpt\text{-}dqd \quad \text{.....................} \quad (26)$$

<u>Free Power Turbine</u>

$$dp4 = D1\ dns+D2\ dt4+D3\ dmaf \quad \text{.......} \quad (27)$$

$$dqfpt = F1\ dns+F2\ dt4+F3\ dmaf \quad \text{.....} \quad (28)$$

<u>High Pressure Turbine</u>

$$de = (dmf\text{-}de)/T \quad \text{........................} \quad (29)$$

from continuity-- $$dmaf = K1\ dmf + K2\ dma \quad \text{................} \quad (30)$$

$$dt4 = G1\ dma + G2\ dt2 + G3\ dp4 + G4\ dng + G5\ de \quad \text{..........} \quad (31)$$

$$dp2 = H1\ dma + H2\ dt2 + H3\ dp4 + H4\ dng + H5\ de \quad \text{.........} \quad (32)$$

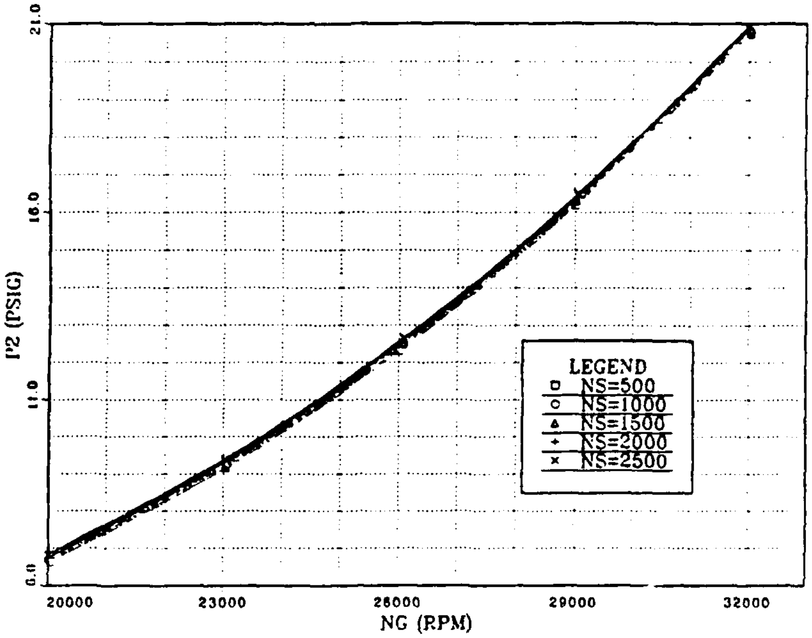
$$dqhpt = I1\ dma + I2\ dt2 + I3\ dp4 + I4\ dng + I5\ de \quad \text{.........} \quad (33)$$

These resulting equations from table 5 will be used to develop the state space relationships through a reduction methodology to get three equations (one each for ng, ns, and e) in terms of ng, ns, e, mf, and qd.

F. Decoupling the Dynamic Equation Set

In the past, during the equation set reduction, a number of the table 3. equations were used more than once. This resulted in the development of singularity points in the simulations. To avoid this, another method was developed in the present work. This was achieved by inspecting the system block diagram (fig. 1) and deciding that if two equations, one for p2 and the other for p4, could be expressed in terms of the states ng, and ns only, then it would be possible to reduce the equation set directly (without resubstitution) thus eliminating the singularities. The new p2 and p4 equations were developed with the assistance of the Minitab regression routine and steady state data for ng and ns. This resulted in the conclusion that both variables had a second power effect with ng and a first power effect with ns. With these relationships in mind, the computer routine was used to fit the data in a two step process. First , equations for p2 and p4 were generated in terms of ng^2, ng, and ns. Next, the data generated from these equations for p2 and p4 was plotted versus ng at five different constant ns values. These plots were compared to the actual data to check for accuracy of the equations. The results are shown on the two graphs in figure 6 and show the robustness of these equations over the entire range of the

P2 VS NG (NS=500~2500 RPM)



P4 VS NG (NS=500~2500 RPM)

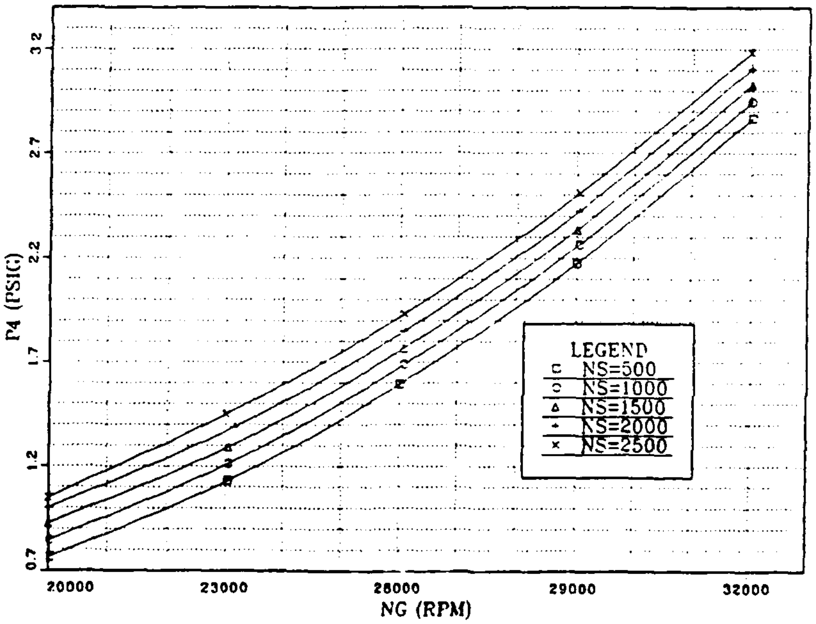


Figure 6. P2 and P4 graphical Representations.
Note: Solid lines are equations, symbols are data.

data. The new dynamic equations for p2 and p4 are now in the form of:

$$p2 = c1\ ng^{\wedge 2} + c2\ ng + c3\ ns + c4 \quad \text{and} \qquad (34)$$

$$p4 = c5\ ng^{\wedge 2} + c6\ ng + c7\ ns + c8. \qquad (35)$$

These equations were then linearized into the form:

$$dp2 = H1\ dng + H2\ dns \qquad \text{for eqn. 11 and} \quad (36)$$

$$dp4 = D1\ dng + D2\ dns \qquad \text{for eqn. 6.} \qquad (37)$$

These twelve equations were then reduced with the reduction methodology as described in detail in Appendices A and B.

The results of the reduction transformed the linearized equations to state space equations, which were used in the simulation model. The resulting state space equations from Appendix B are:

$$dng = \frac{Z1}{JGB}\, dng + \frac{Z2}{JGB}\, dns + \frac{Z3}{JGB}\, de$$

$$dns = \frac{Z4}{JDB}\, dng + \frac{Z5}{JDB}\, dns + \frac{Z6}{JDB}\, de + \frac{Z7}{JDB}\, dmf - \frac{1}{JDB}\, dqd$$

$$de = -\frac{1}{T}\, de + \frac{1}{T}\, dmf$$

where Z1 through Z7 were derived from combinations of the A1 through K2 coefficients of the dynamic equations. The standard state space equation is $\underline{x} = \underline{A}(t)\ x + \underline{B}(t)\ u$.[9] Here $\underline{x}$ is a matrix made up of dng, dns, and de, x is a state vector composed of the state variables: dng, dns, and de, and u is a vector composed of inputs dmf and dqd.

The A and B matrices are:

$$A11 = \frac{Z1}{JGB} \qquad A12 = \frac{Z2}{JGB} \qquad A13 = \frac{Z3}{JGB}$$

$$A21 = \frac{Z4}{JDB} \qquad A22 = \frac{Z5}{JDB} \qquad A23 = \frac{Z6}{JDB}$$

$$A31 = O \qquad A32 = O \qquad A33 = \frac{-1}{T}$$

and

$$B11 = O \qquad B12 = O$$

$$B21 = \frac{Z7}{JDB} \qquad B22 = \frac{-1}{JDB}$$

$$B31 = \frac{1}{T} \qquad B32 = O$$

and the state space equations now appear as:

$$dng = A11\ dng + A12\ dns + A13\ de + B11\ dmf + B12\ dmf$$

$$dns = A21\ dng + A22\ dns + A23\ de + B21\ dmf + B22\ dmf$$

$$de = A31\ dng + A32\ dns + A33\ de + B31\ dmf + B32\ dmf.$$

A coarse tuning was next performed to determine the best numerical values of the coefficients. This was necessary since the first set of regressed coefficients did not provide accurate simulation. This was attributed to the coarse instrumentation that is present on the NPS turbine and the manner of fitting in the regression routine. Consequently, attention was focused on the following coefficients as in need of tuning: A1, A2, B1, B2, C1, C2, F1, and I1 - I5 . Those for D1, D2, H1, H2, K1, and K2 were found previously using the data regression program. Initially chosen for a starting point was that for B2 ($\partial ma/\partial p2$). This was an obvious choice for much is known about the relationship between ma and p2 in compressors. To find B2, p2 was plotted versus ma at constant ng values. From this the slope $\partial ma/\partial p2$ (B2) was obtained. The B1 expression ($\partial ma/\partial ng$) was found by regression using the plotted value of B2 in the Minitab routine. A2 and C2 were then found using plotted values to find the slopes $\partial t2/\partial p2$ (A2) and $\partial qc/\partial p2$ (C2). These were then used as inputs into the relationships:

$$\frac{B1}{B2} = \frac{\partial ma/\partial ng}{\partial ma/\partial p2} = \frac{A1}{A2} = \frac{\partial t2/\partial ng}{\partial t2/\partial p2} = \frac{C1}{C2} = \frac{\partial qc/\partial ng}{\partial qc/\partial p2}$$

from which A1 and C1 were obtained. The results were then checked and verified through the use of the data regression program. At steady state qc = qhpt is required, thus the relationship for C1 ($\partial qc/\partial ng$) = I4 ($\partial qhpt/\partial ng$). So, with I4 determined, the value of I1 ($\partial qhpt/\partial ma$) can be found as I1 = I4/B1 and I2 ($\partial qhpt/\partial t2$) is I4/A1. With these three, I1, I2, and I4, as inputs, I3 ($\partial qhpt/\partial p4$) and I5 ($\partial qhpt/\partial mf$) were obtained through the use of the data regression routine. The numerical values can be found in Appendix C.

The next step involves fine tuning the computer simulation model and results in the setting of the remaining coefficients of the A and B matrices.

IV. FINE TUNING THE MODEL

In fine tuning, the actual dynamic data was used in conjunction with a simulation approach to select final values for the model constants.

The simulation approach used the simple linear state space equations for dng, dns, and de. These described a first order differential equation set, with time varying A and B matrices, to mimic the actual nonlinearity of the system response. This was accomplished by approximating the true nonlinear relationships with a series of small linear steps as depicted in figure 7.

The dynamic simulation computer model was developed from a routine written by Herda and later modified by Stammetti, it is shown in figure 8. The major change to the previous program was in the A and B matrix equations. In the past, the equations were complex and were a combination of exponentials and second order equations. In the present model they were simple, first order linear functions of the states. Another change which occurred in the fine tuning process was the adjusting of the dynomometer shaft inertia. This change was for shaping the response and did not affect the steady state response of the model.

A. Computer Simulation

The simulation program was built using four sections. These were

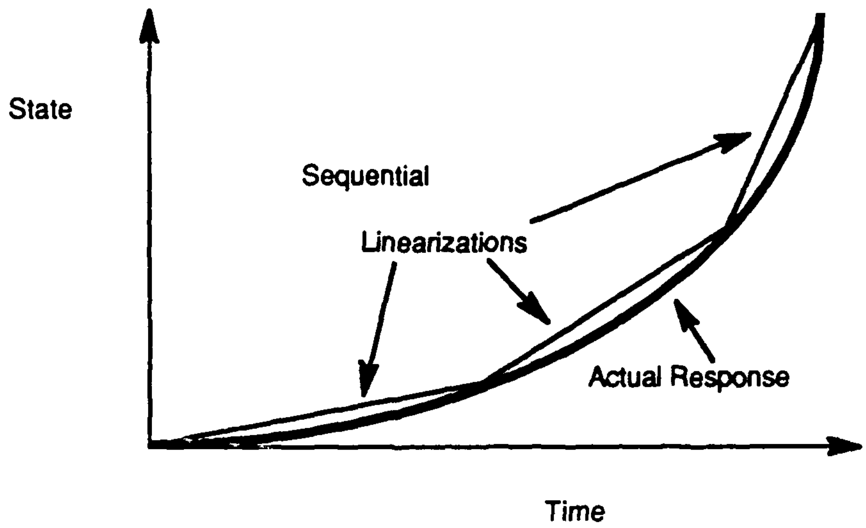


Figure 7. Sequential Linearization.

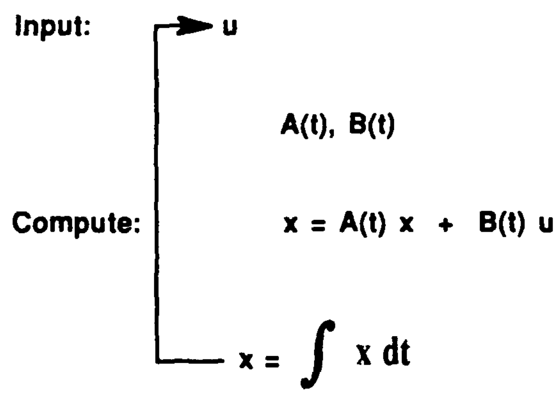


Figure 8. Simulation Process.

the dynamic data input, the initial condition, the dynamic, and the derivative sections. The actual dynamic data was taken on a four channel strip chart recorder and depicts the real turbine response of Ng, Ns, Qd, and Mf as a function of time. These dynamic runs were compiled in Appendix E and provided the source and data for the simulation. The initial condition section established the initial conditions for each particular run. The dynamic section formulated the display of the dynamic data and was followed by the derivative section which formulated the simulated response. The outputs of these final two sections were then plotted for comparison. These plotted results were used to assist in a grid search on the still doubtful component coefficients to determine the final values of the A and B matrices. The procedure was to guess values of the coefficients, then simulate the model response. If the model and the data agreed, then the search was over.

B. The Grid Search

A grid search technique was applied in two and three parameter searches to find the optimal values for the remaining coefficients. It was formed using a the spread sheet formatted program called Excell, by Microsoft software. An example of this is found in Appendix C. The top portion provided the inputs for the coefficients used in the calculation of the A and B matrices. The middle portion calculated the intermediate Y variables. These were then combined with the previously mentioned A thru K coefficients to calculate the A and B matrix values, three of which were ng dependent.

We also knew that the steady state required x = 0. This fact was used to generate additional equations which must be satisfied by any coefficient set to guide the grid search. The last section of the spread sheet provided for the calculation of the values of dng and dns for the simulation runs 3 and 9 as well as the peak dns value for run three (figure 9). Run three, shown in figure 9, developed into the most difficult run for determining the best coefficients for Ns simulation and is the reason for the addition of the peak dns value. The values for the G, F2, and F3 coefficients were the focus of the grid search as we had least confidence in them. These are the important coefficients in determining the value of dns and in turn, the ns simulation values. The G1 - G5 values were calculated in the same manner as described eariler for the I1- I5 values. They represent the partial derivatives of the equation for t4 (eqn. 10) and were determined to have little effect on the A21, A22 and A33 results. Their elimination as a search parameter reduced the search to a two parameter one. Various values for F2 and F3 were tried, and the resulting A and B matrix coefficients calculated were inserted in the computer simulation. The results were plotted on the NPS mainframe computer system, using the easyplot routine, to check for accuracy of simulation. This process was

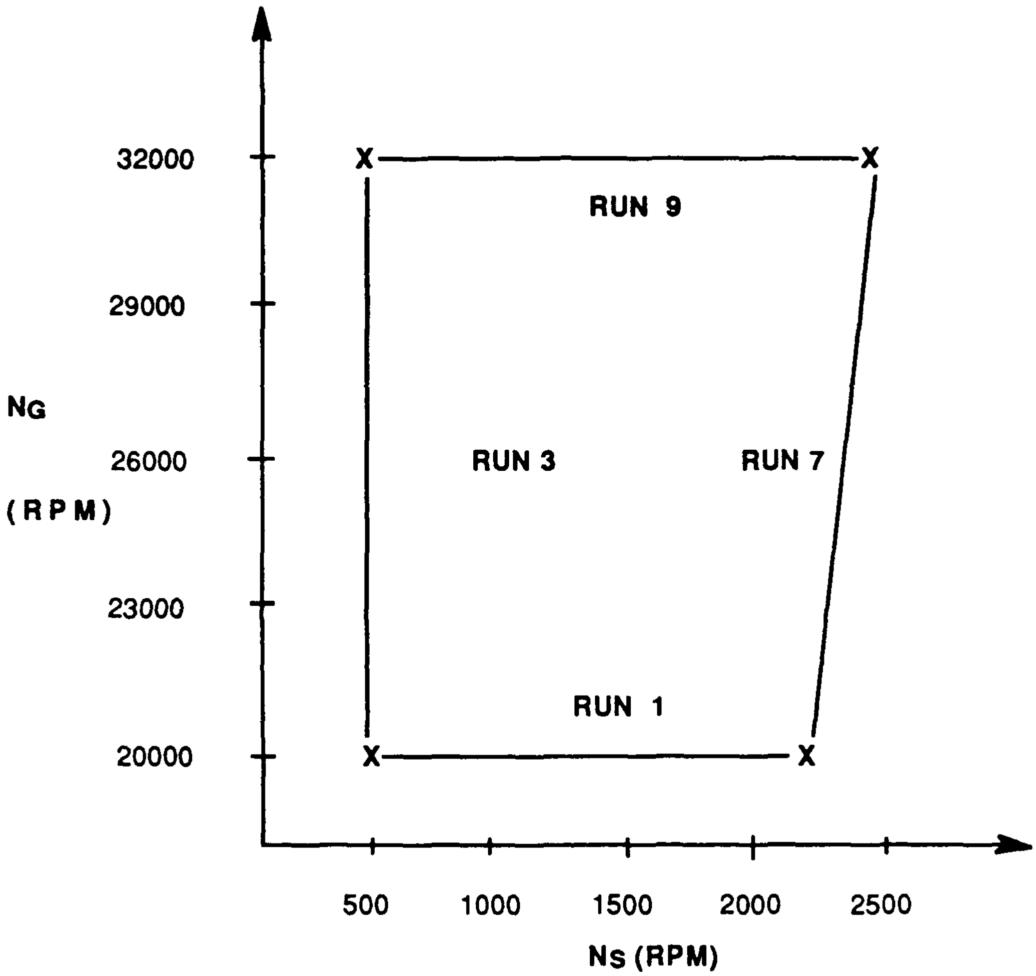


Figure 9. Dynamic Data Runs.

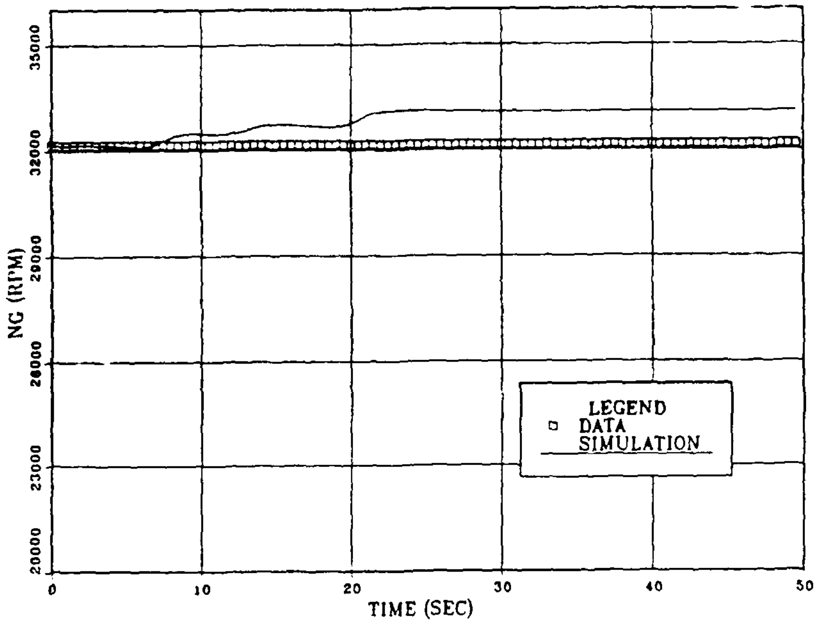
repeated until the optimal F2 and F3 values were obtained.

V. VALIDATION OF THE MODEL

The validation of the model was a two step process which carried over from the fine tuning. After the fine tuning was complete, and the model resulted in the proper shape for runs 3 and 9, and with the steady state response within the required tolerances, the first step was complete. The next step was to take the model and apply it elsewhere in the operating range of the turbine to check for accuracy. If the proper response in shape and prescribed tolerance criteria (+/ - 10 percent in the dynamic and steady response) were still met, then the validation was completed. This process was only performed around the borders of the operating range (figure 9). If successful, then the model should be applicable anywhere in the turbine operating range.

The first run modeled was the one for run nine which models the Ns transient response from 500 to 2500 rpms while holding the value for Ng, as load varies, constant at 32000 rpms (+/- 5%) for the conditions shown in run nine in Appendix E.. Figure 10 is the graphical representation of the modeled run and the dynamic data run for this transient. The criteria has been met for run 9, so the model was applied to the validation run one. Run one, simulates the transient for Ns from 500 to 2215 rpms (the turbine will not operate above 2215 rpm at low Ng) with the value for Ng constant at 20000

SIMULATION RUN 9 NG



SIMULATION RUN 9 NS

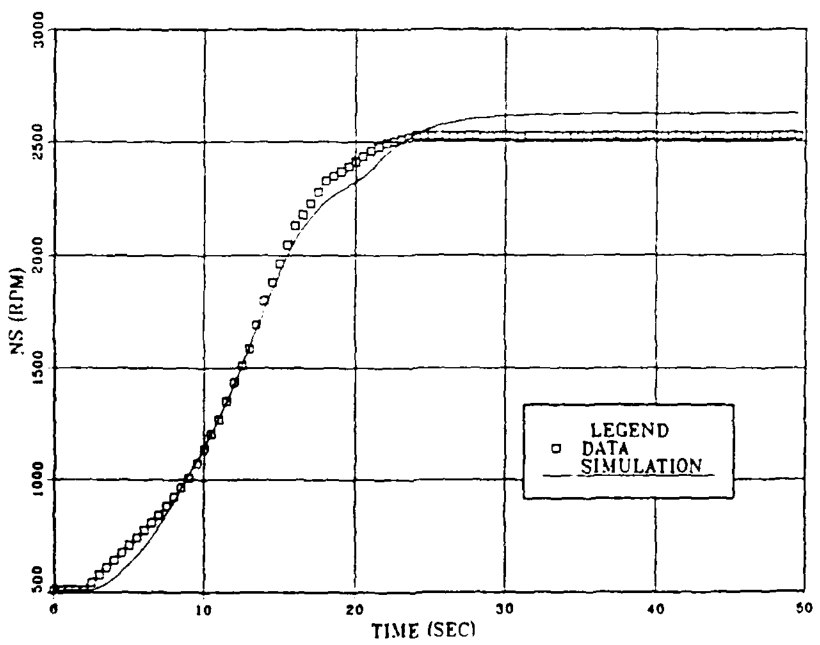
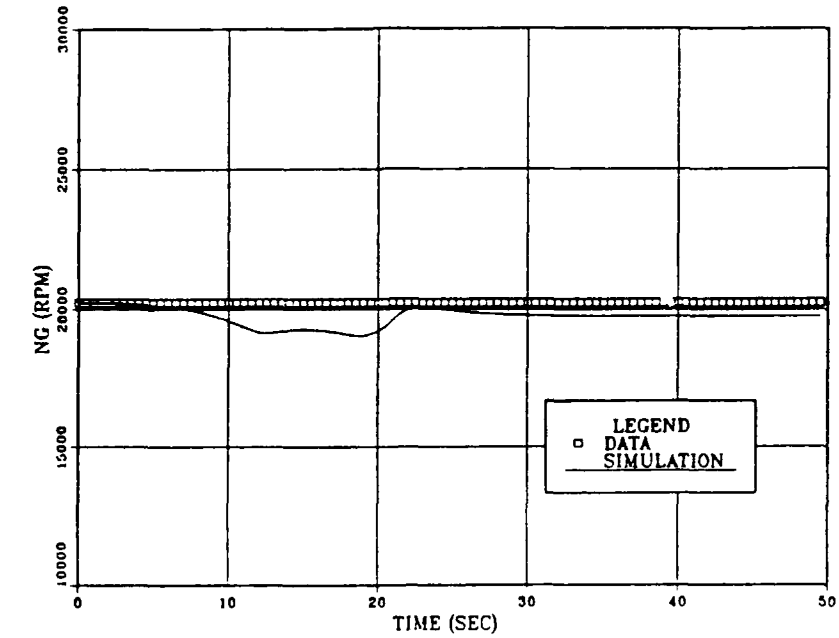


Figure 10. Simulation-Run Nine.

VALIDATION RUN 1 NG



VALIDATION RUN 1 NS

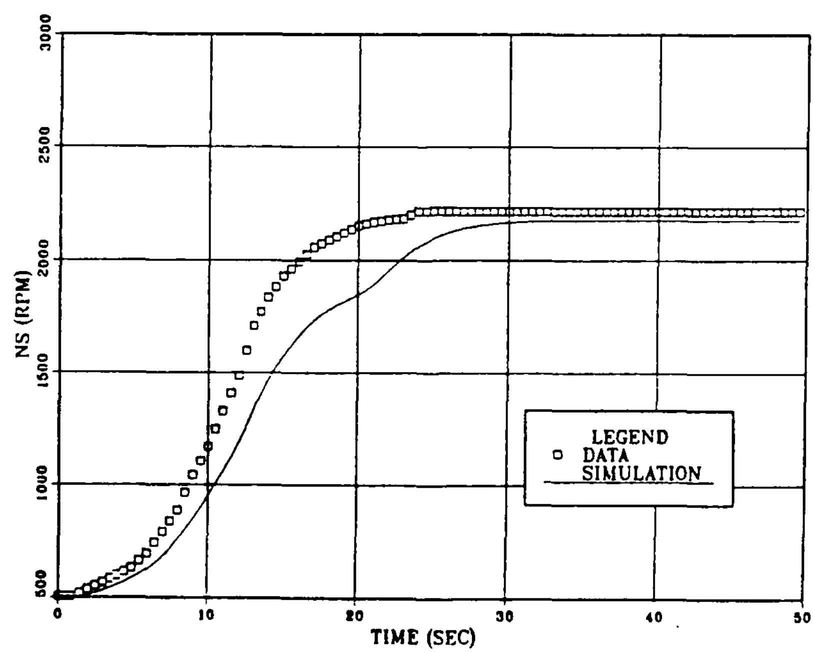
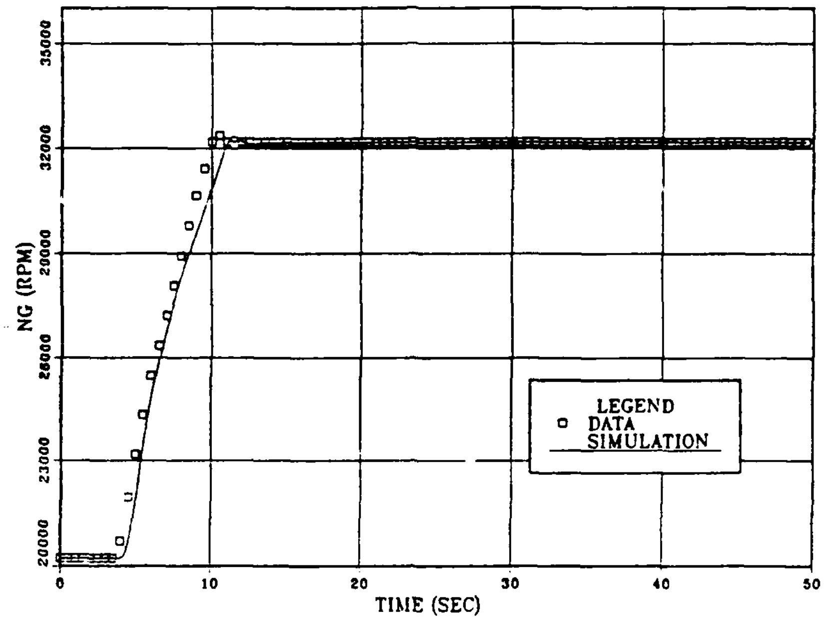


Figure 11. Validation-Run One.

rpms (+/- 5%); the results are shown in figure 11. Again, the accuracy criteria were met. This covers the upper and lower bounds of the operating range.

Next, run three was used in conjunction with run nine in the fine tuning portion. This run is a vertical run on the left side of the operating range and is an Ng transient from 20000 to 32000 rpms with Ns being held constant at 500 rpms to conform with the actual dynamic data run. This run is shown in figure 12. Note that the accuracy criteria are easily met in Ng but apparently not satisfied in Ns. However, when the accuracy criteria is applied to the normalized Ns, the transient is acceptable. To validate this run the model is next applied to the right hand side of the operating range. This is run seven and, again, is a Ng transient from 20000 to 32000 rpms with Ns slightly increasing from the lower constraint of 2215 rpm to 2500 rpms which is attainable at the upper limits of the operating range. This run is presented in figure 13. A validation run was attempted vertically in the center of the operating range. This is run five and it's response was of the same result shown in figure 11 for run seven and therefore was not shown as a separate figure.

SIMULATION RUN 3 NG



SIMULATION RUN 3 NS

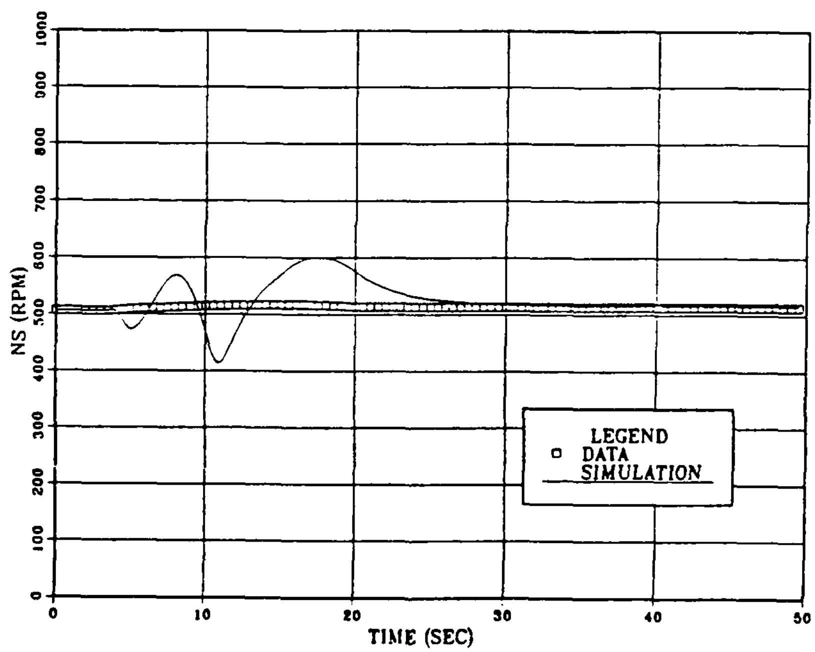
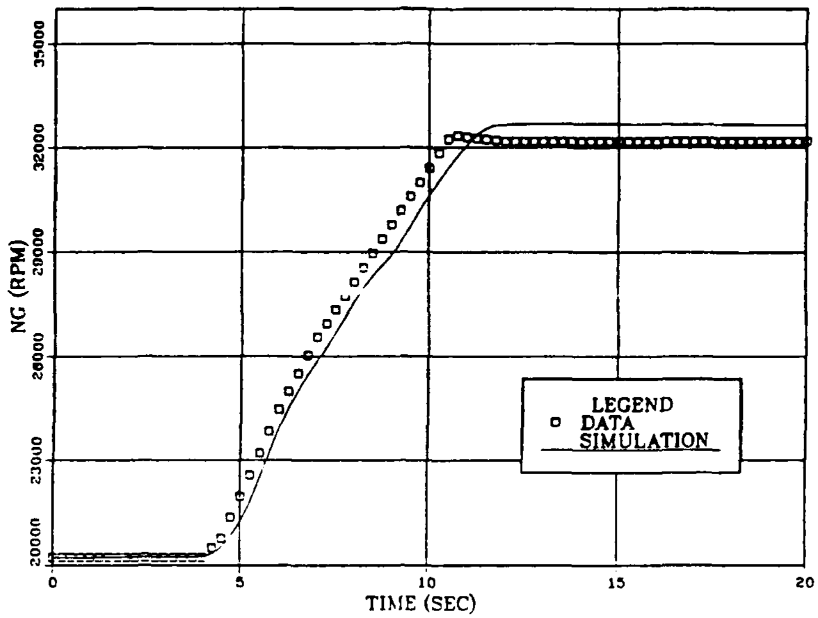


Figure 12. Simulation-Run Three.

VALIDATION RUN 7 NG



VALIDATION RUN 7 NS

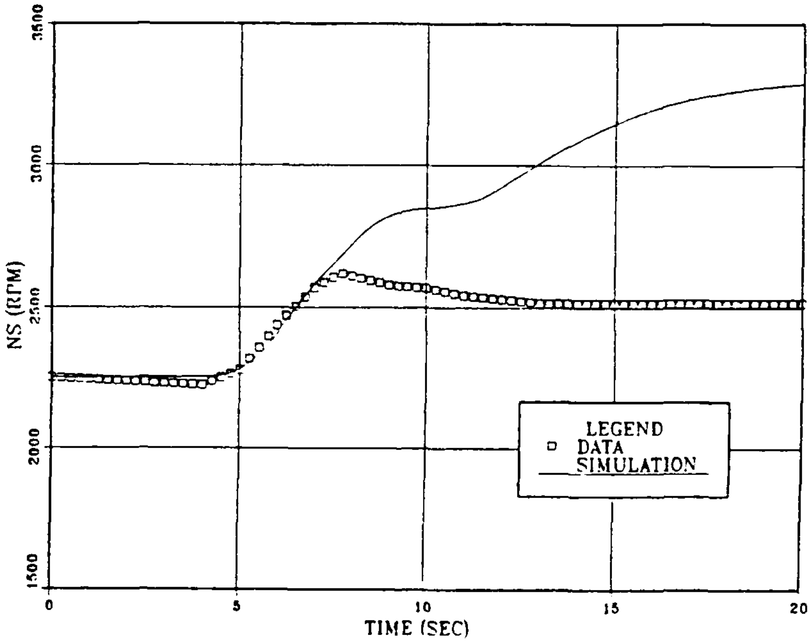


Figure 13. Validation-Run Seven.

VI. CONCLUSIONS

A modeling method has been developed for real time simulation of a marine gas turbine. The results of this modeling method should be used with the LQR controller design theory to develop a new controller for the Boeing test bed at NPS. This controller should then be tested to ascertain it's ability to be more fuel efficient than the present system.

This model was simple in that it used matrix elements for A and B which were only linear state functions. The dependency was only on the Ng state. This simplicity will provide fast run times on a small computer for model reference control and diagnostics.

Accurate simulation performance has been demonstrated in the compressor/high pressure turbine section and the resulting steady states have been achieved to +/- 3%. An exception to this accuracy is the constant Ns simulation for the vertical runs . This is only an indication that the final values for F2, F3, and possibly G1 are not fine tuned enough to provide for more accurate constant Ns response. Investigation was conducted into this irregularity to determine the cause of the steady state results being to high. The fact that the steady state response for a constant Ns is higher in runs five and seven was probed and a number of solutions were looked at. One such solution was to check for the possibility that the oscillating torque for runs five and seven shown in Appendix E was causing the Ns value to continue to rise and settle out at a higher steady state. These oscillations do not appear to be the cause nor have any affect on the steady state response of Ns. This was verified by modifying the computer simulation routine so as to remove the oscillation. Another solution was to adjust the A21 values until a proper steady state value for ns was reached. At the time of publication this has not provided any satisfactory results. Another possibility is that the A21 coefficients may have to change the sign to produce the proper response. The last area to examine as a possible solution to this problem is to input a varying inertia for the water brake dynamometer. In the past this inertia has been assumed to be a constant value when, in fact, it actually changes with the addition and subtraction of water.

The last recommendations for this project encompass the entire project. First, a digital data acquisition system should be used to get the steady state and dynamic data. This will make the fine tuning process with the data regression program more accurate. Finally, one computer system should be used for the whole process, from the regression and grid search to the computer simulation (vice the two system used in this project). In this way, results could be achieved and correlated in a more efficient manner.

The model was robust in that, with the exception of the constant Ns in the vertical runs, the model was applicable to all areas of the operating range. This was supported by the excellent response in Ng throughout the whole operating range. This shows that the coefficients for the A11, A12, and A13 were accurate and the procedure to find them was sound.

Once the proper values for A21, A22, and A23 are found and this modeling technique has been successfully demonstrated on the Boeing engine this method will have real time application to the future controller design for the General Electric LM- 2500 and the follow on marine gas turbine controllers designed for the future.

APPENDICES

APPENDIX A: COMPONENT MODELING EQUATIONS

APPENDIX B: EQUATION SET REDUCTION METHODOLOGY

APPENDIX C: PARAMETER SELECTION METHODOLOGY

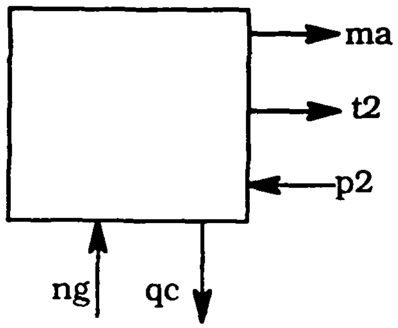
APPENDIX D: COMPUTER SIMULATION CODES

APPENDIX E: SOURCE DATA RUNS

APPENDIX A

COMPONENT MODELING EQUATIONS :

Compressor

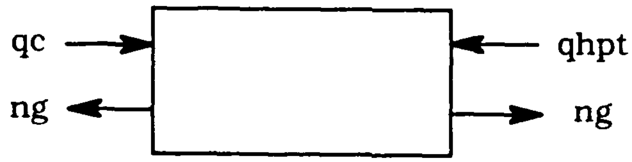


$$dt2 = A1\ dng + A2\ dp2 \quad \ldots\ldots\ldots\ldots \quad 1$$

$$dma = B1\ dng + B2\ dp2 \quad \ldots\ldots\ldots\ldots \quad 2$$

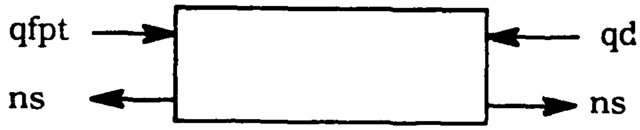
$$dqc = C1\ dng + C2\ dp2 \quad \ldots\ldots\ldots\ldots \quad 3$$

Gas Generator Shaft



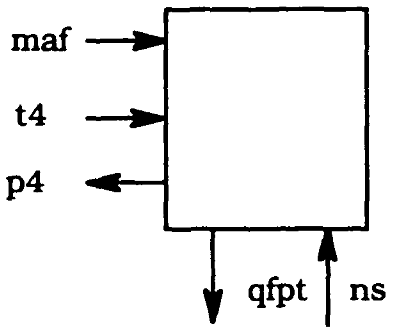
$$JGB\ dng = dqhpt - dqc \quad \ldots\ldots \quad 4$$

Dynamometer Shaft



$$JDB\ dns = dqfpt - dqd \quad \ldots\ldots \quad 5$$

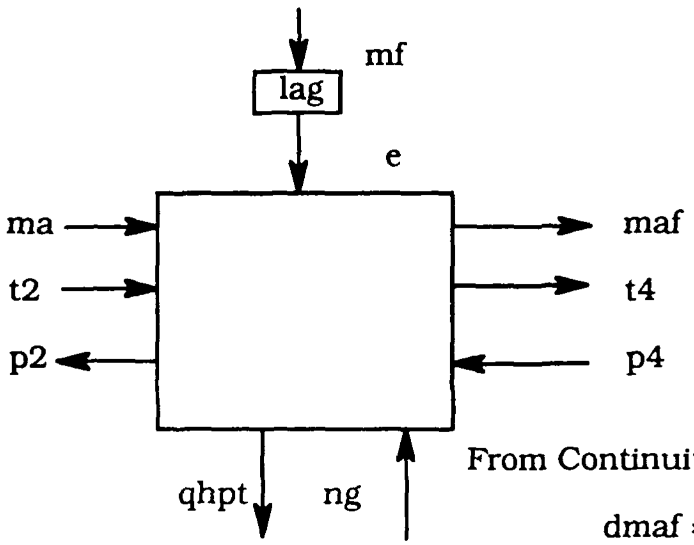
Free Power Turbine



$dp4 = D_1\ dng + D_2\ dns$ .... 6 *

$dqfpt = F_1\ dns + F_2\ dt4 + F_3\ dmaf$ ... 7

High Pressure Turbine



$de = (dmf - de)/T$ 8

From Continuity:

$dmaf = K_1\ dmf + K_2\ dma$ 9

$dt4 = G_1\ dma + G_2\ dt2 + G_3\ dp4 + G_4\ dng + G_5\ de$ 10

$dp2 = H_1\ dng + H_2\ dns$ .... 11 *

$dqhpt = I_1\ dma + I_2\ dt2 + I_3\ dp4 + I_4\ dng + I_5\ de$ 12

Note: 1. Lower case variables represent normalized variables.

2. The d _ represents the perturbation of the variable.

* Decoupling equations.

APPENDIX B

EQUATION SET REDUCTION METHODOLOGY:

The equations in appendix A are reduced into the state space components of the A and B matrices.

Substitute eqn 11:

dp2 = H1 dng + H2 dns 11

into eqns 1, 2, and 3 for dp2.

dt2 = (A1 + A2H1) dng + (A2H2) dns 1

dma = (B1 + B2H1) dng + (B2H2) dns 2

dqc = (C1 + C2H1) dng + (C2H2) dns 3

In eqn 12:

dqhpt = I1 dma + I2 dt2 + I3 dp4 + I4 dng + I5 de 12

substitute eqns 1, 2, and 6 for dt2, dma, and dp4.

dqhpt = I1 (B1 + B2H1) dng + I1 (B2H2) dns +

I2 (A1 + A2H1) dng + I2 (A2H2) dns +

I3 (D1) dng + I3 (D2) dns +

I4 dng + I5 de 12

Collect terms:

$$dqhpt = (I1\ (B1 + B2H1) + I2\ (A1 + A2H1) + I3\ (D1) + I4\ )\ dng +$$

$$(I1\ (B2H2) + I2\ (A2H2) + I3\ (D2)\ )\ dns + I5\ de$$

Let $Y1 = I1\ (B1 + B2H1) + I2\ (A1 + A2H1) + I3\ (D1) + I4$

$Y2 = I1\ (B2H2) + I2\ (A2H2) + I3\ (D2)$

Then: $dqhpt = Y1\ dng + Y2\ dns + I5\ de$ 12

Take eqn 4 and substitute in new eqn 12 for dqhpt and eqn 3 for dqc.

$$JGB\ dng = Y1\ dng + Y2\ dns + I5\ de\ -$$

$$(C1 + C2H1)\ dng - (C2H2)\ dns$$

Collect terms:

$$dng = (\ (Y1 - (C1 + C2H1))\ dng + (Y2 - (C2H2))\ dns + I5\ de\ )/JGB$$

Let: $Z1 = (Y1 - (C1 + C2H1)$

$Z2 = Y2 - (C2H2)$

$Z3 = I5$

Then:

$dng = (Z1\ dng + Z2\ dns + Z3\ de)/JGB$ R1

In eqn 10:

$$dt4 = G1\ dma + G2\ dt2 + G3\ dp4 + G4\ dng + G5\ de \quad \ldots\ldots\ldots\ldots\ 10$$

substitute eqns 1, 2, and 6 for dt2, dma, and dp4.

$$\begin{aligned} dt4 = {} & G1\ (B1 + B2H1)\ dng + G1\ (B2H2)\ dns + \\ & G2\ (A1 + A2H1)\ dng + G2\ (A2H2)\ dns + \\ & G3\ (D1)\ dng \qquad + G3\ (D2)\ dns + \\ & G4\ dng \qquad + G5\ de \end{aligned} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 10$$

Collect terms:

$$\begin{aligned} dt4 = {} & (G1\ (B1 + B2H1) + G2\ (A1 + A2H1) + G3\ (D1) + G4\ )\ dng + \\ & (G1\ (B2H2) + G2\ (A2H2) + G3\ (D2)\ )\ dns + G5\ de \end{aligned}$$

Let $Y3 = G1\ (B1 + B2H1) + G2\ (A1 + A2H1) + G3\ (D1) + G4$

$Y4 = G1\ (B2H2) + G2\ (A2H2) + G3\ (D2)$

Then: $dt4 = Y3\ dng + Y4\ dns + G5\ de \quad \ldots\ldots\ldots\ldots\ldots\ldots\ 10$

In the continuity eqn (9) substitute eqn 2 for dma.

$$dmaf = K1\ dmf + K2(B1 + B2H1)\ dng + K2(B2H2)\ dns \quad \ldots\ldots\ldots\ 9$$

Substitute new eqn 10 for dt4 and new eqn 9 for dmaf into:

eqn 7: $dqfpt = F1\ dns + F2\ dt4 + F3\ dmaf$

This results in:

$$dqfpt = F1\ dns + F2Y3\ dng + F2Y4\ dns + F2G5\ de +$$

$$F3K1\ dmf + F3K2(B1 + B2H1)\ dng + F3K2(B2H2)\ dns$$

Collect terms:

$$dqfpt = (F2Y3 + F3K2(B1 + B2H1))\ dng +$$

$$(F1 + F3K2(B2H2) + F2Y4)\ dns + F2G5\ de + F3K1\ dmf$$

Let: $Z4 = F2Y3 + F3K2(B1 + B2H1)$

$Z5 = F1 + F3K2(B2H2) + F2Y4$

$Z6 = F2G5$

$Z7 = F3K1$

Then:

$$dqfpt = Z4\ dng + Z5\ dns + Z6\ de + Z7\ dmf \quad \text{.............} \; 7$$

Substitute new eqn 7 for qfpt in eqn 5.

$$JDB\ dns = dqfpt - dqd \quad \text{......................................} \; 5$$

$$dns = (Z4\ dng + Z5\ dns + Z6\ de + Z7\ dmf - dqd)/JDB \quad \text{...............} \; R2$$

Finally: Eqn 8 is R3. R1, R2, and R3 make up the state space equation set.

The state space equation set is the following:

$$dng = \frac{Z1}{JGB} dng + \frac{Z2}{JGB} dns + \frac{Z3}{JGB} de \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ R1$$

$$dns = \frac{Z4}{JDB} dng + \frac{Z5}{JDB} dns + \frac{Z6}{JDB} de + \frac{Z7}{JDB} dmf - \frac{1}{JDB} dqd \qquad \ldots\ldots\ R2$$

$$de = \qquad - \frac{1}{T} de + \frac{1}{T} dmf \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ R3$$

The resulting A and B matrices are:

$$A11 = \frac{Z1}{JGB} \qquad A12 = \frac{Z2}{JGB} \qquad A13 = \frac{Z3}{JGB}$$

$$A21 = \frac{Z4}{JDB} \qquad A22 = \frac{Z5}{JDB} \qquad A23 = \frac{Z6}{JDB}$$

$$A31 = O \qquad A32 = O \qquad A33 = \frac{-1}{T}$$

and

$$B11 = O \qquad B12 = O$$

$$B21 = \frac{Z7}{JDB} \qquad B22 = \frac{-1}{JDB}$$

$$B31 = \frac{1}{T} \qquad B32 = O$$

APPENDIX C

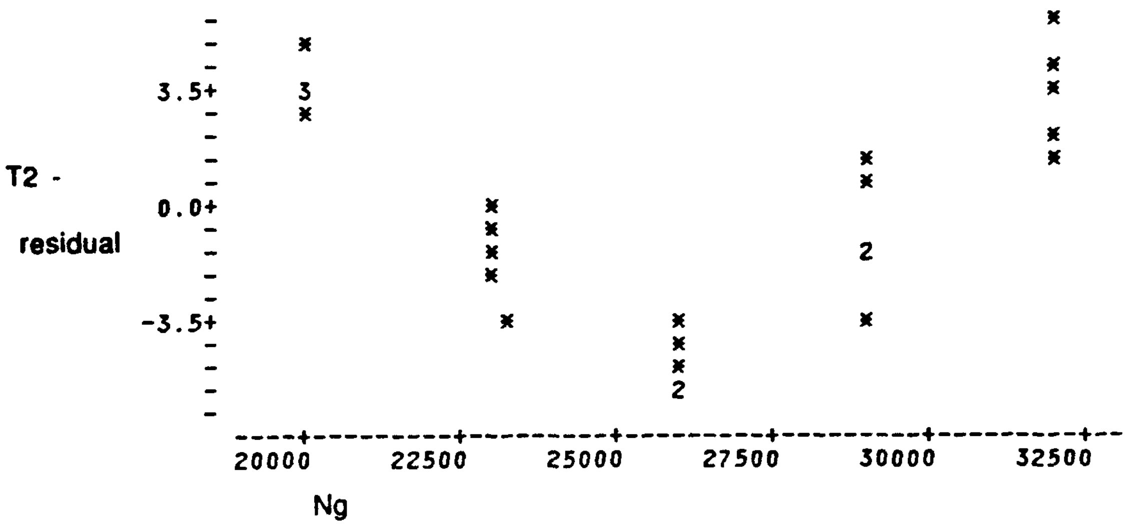
PARAMETER SELECTION METHODOLOGY:

The parameter selections were accomplished through three means:

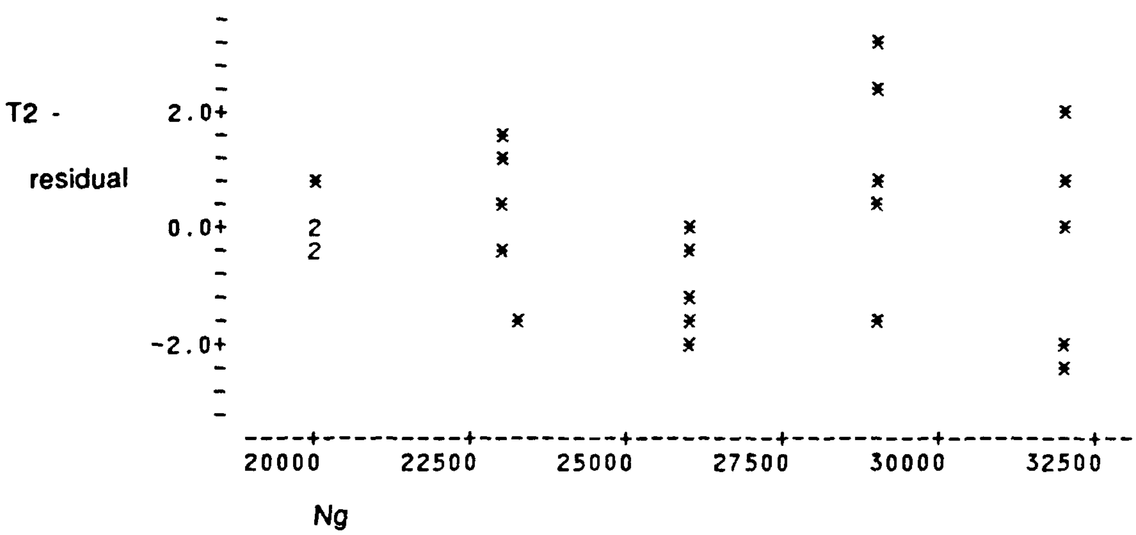
1. Plotting actual slopes from data. This was shown in chapter III section F.
2. Data reduction using mainframe program Mini-Tab.
3. Grid searching using coefficients from 1. and 2. and known relationships which must be satisfied. The spread sheet is from the Micro-soft program Excell and was used on an Apple Macintosh computer.

The data reduction program Minitab was used for all of the regressions performed. The input for the regression program was the steady state turbine data shown in table 1 A. This routine was used to determine the order of the independent variables in the equations for Ma, T2, and Qc, as well as to regress the expressions for P2, P4, T4, Qhpt and Maf. To determine the order of a independent variable a three step process was followed. First, the dependent variable was regressed using the data for the independent variables. In doing this a residual was calculated for each set of the independent variables. This was then subtracted from the regressed value of the dependent variable. The resulting difference was plotted versus each of the independent variables. In the plot for the regression of T2, shown in figure C-1a, there is a definite parabolic shape which is an indication that the independent variable in question (Ng) is of a higher order. To verify this, T2 was regressed again, this time using Ng^2, Ng and P2. The plotting was repeated with the resultant plot shown in figure C-1b. This time the parabolic shape was not in evidence and the predictor percentage, which indicates the accuracy of the regression based on the data, increased from 99.6 to 99.9 percent indicating an excellent regression for the t2 equation. The resulting partials for all of the component expressions are shown in figure C-2 lines 3 through 21.

The spreadsheet grid, figure C-2, was used in conjunction with the simulation program to search out the best values for the rest of the coefficients in the equations in appendix A. These were input for G1, F2, and F3 in lines 3-21 and then used in lines 25-32 to calculate the values of the coefficients of the A and B matrices. Lines 34-38 show the proper signs for these matrix coefficients as well as the dns and dng relationships which must be satisfied during the search process. Once these were satisfied, plots were generated and checked for the accuracy of the simulation.



(A)



(B)

Figure C-1. Regression Plots

	A	B	C	D	E	F	G
3							
4		A	A X NG	B	B X NG	C	C X NG
5	1	-2.6	13.15	-1.06	5.36	-2.415	12.21
6	2	-3.5		-1.427		-3.25	
7							
8							
9		D	D X NG	F	F X NG	K	
10	1	-1.97	4.36	-0.06	-0.445	0.0141	
11	2	0.039		-1.55		0.975	
12	3			2.5			
13							
14							
15							
16		G	G X NG	H	H X NG	I	I X NG
17	1	0.020335		-1.44	3.62	2.278	
18	2	0.00829111		0.0134		0.9288	
19	3	-0.0925698				-10.37	
20	4	-0.021558	0.10899489			-2.415	12.21
21	5	0.12161723				13.624	
22							
23							
24		Y	Y X NG		Z	Z X NG	
25	1	22.5465086	-32.114852		20.2815086	-32.559852	A11
26	2	-0.4915502			-0.4480002		A12
27	3	0.20126569	-0.2866793		13.624		A13
28	4	-0.0043879			2.11305817	0.91786171	A21
29	5				-0.0998081	-0.445	A22
30	6	RUN 3 ng =	1.23769231		-0.1885067		A23
31	7	RUN 9 ng =	1.23653846		0.03525		B21
32		PEAK ng =	1.23630769				
33							
34	RUN 3 dng =	-0.0001131	dns =	0.00094893	- A11	- A12	+ A13
35		----------		----------			
36	RUN 9 dng =	0.25454755	dns =	-0.0113783	+ A21	- A22	+ A23
37		----------		----------			
38			PEAK dns =	0.12769172	0	0	- A33
39							
40							

Figure C-2. Sample Grid Search Spread Sheet.

APPENDIX D

COMPUTER SIMULATION CODES:

The computer codes were written using the Dynamic Simulation Language (DSL).

Simulation Run 1: DSL code for dynamic Ns run at constant Ng (20000 rpm).

Simulation Run 3: DSL code for dynamic Ng run at constant Ns (500 rpm).

Simulation Run 5: DSL code for dynamic Ng run at constant Ns (1500 rpm).

Simulation Run 7 : DSL code for dynamic Ng run at Ns (2252-2500 rpm).

Simulation Run 9: DSL code for dynamic Ns run at constant Ng (32000 rpm).

```
************************************************************************
*                                                                      *
*                                                                      *
*                  BOEING MODEL 502-6A GAS TURBINE                     *
*                                                                      *
*                    DYNAMIC COMPUTER SIMULATION                       *
*                                                                      *
*                           MODIFIED BY                                *
*                            S.D. METZ                                 *
*                        FROM  ROUTINES BY                             *
*                  V.J.HERDA AND V.A. STAMMETTI                        *
*                                                                      *
*        THIS PROGRAM SIMULATES THE DYNAMIC RESPONSE OF THE NPS        *
*    BOEING GAS TURBINE TEST FACILITY USING A MULTILPLE                *
*    LINEARIZATION TECHNIQUE.                                          *
*                                                                      *
************************************************************************
C
C
PARAM JG=0.009525, JD=0.3000, PI=3.14159, TC=1.0
*
*   THE FOLLOWING VALUES LISTED ON THE FUNCTION CARD ARE FOR FUEL FLOW,
*   GAS GENERATOR SPEED, TORQUE AND DYNO SPEED AS A FUNCTION OF TIME.
*   THESE VALUES WERE OBTAINED FROM STRIP CHART RECORDS AND ARE ENTERED
*   IN THE FORM (E.G. FUEL FLOW)  ...TIME(SEC), FUEL FLOW.....
*
*
C
C        THIS SET IS FOR EXPERIMENTAL RUN # 1.
C
C
AFGEN NGDATA= 0.0,20230.0, 5.0,20230.0, 10.0,20230.0, 15.0,20230.0, ...
         20.0,20230.0, 25.0,20230.0, 50.0,20230.0
AFGEN NSDATA= 0.0,505.0, 1.0,505.0, 2.0,536.7, 3.0,568.33, ...
         4.0,600.0, 5.0,631.7, 6.0,695.0, 7.0,790.0, 8.0,885.0, ...
         9.0,1043.3, 10.0,1170.0, 11.0,1328.3, 12.0,1486.7, ...
         13.0,1708.3, 14.0,1835.0, 15.0,1930.0, 16.0,1993.3, ...
         17.0,2056.7, 18.0,2088.3, 19.0,2120.0, 20.0,2151.7, ...
         21.0,2167.5, 22.0,2177.0, 23.0,2183.3, 24.0,2215.0, ...
         25.0,2215.0, 50.0,2215.0
AFGEN MFDATA= 0.0,78.39, 5.0,78.39, 10.0,78.39, 12.0,79.89, ...
         15.0,80.26, 17.0,80.26, 19.0,81.38, 20.0,82.50, ...
         21.0,82.13, 25.0,82.13, 50.0,82.13


AFGEN QDDATA= 0.0,125.0, 1.0,125.0, 2.0,118.3, 3.0,115.0, ...
         4.0,111.7, 5.0,105.0, 6.0,98.33, 7.0,85.0, ...
         8.0,71.67, 9.0,58.33, 10.0,45.0, 11.0,31.67, ...
         12.0,11.67, 12.5,1.670, 13.0,5.0, 15.0,5.0, ...
         25.0,5.0, 50.0,5.0
*
*
*
INITIAL
*   ESTABLISH INITIAL CONDITIONS.
*
        NGI=20230.0
        NSI=505.0
        MFI = 78.39
        QDI = 125.0
*
*
*
*  SET INITIAL STATE PERTURBATION TO ZERO
*
        DNG = 0.0
        DNS = 0.0
        DE  = 0.0
```

```
          NGB = 26000.0
          NSB = 1500.0
          MFB = 122.5
          QDB = 220.0
          QCB = 80.0
          JGB = (NGB/QCB)*JG
          JDB = (NSB/QDB)*JD
DYNAMIC
*
*         DATA CURVE FORMULATION
*
          NGD = AFGEN(NGDATA,TIME)
          NSD = AFGEN(NSDATA,TIME)
          MFD = AFGEN(MFDATA,TIME)
          QDD = AFGEN(QDDATA,TIME)
        MFDLY = TRANSP(100,MFI,0.70,MFD)
*
*        STATE SPACE LINEAR MODEL FORMULATION
*
       A11 = (20.282-32.560*(NGL/NGB))/(JGB)
*
       A12 = (-0.4480)/(JGB)
*
       A13 = (13.624)/(JGB)
*
       A21 = (2.1131+.91786*(NGL/NGB))/(JDB)
*
       A22 = (-.0998-0.445*(NGL/NGB))/(JDB)
*
       A23 = (-0.1885)/(JDB)
*
       A31 = 0.0
*
       A32 = 0.0
*
       A33 = -1.0/TC
*
       B11 = 0.0
*
       B12 = 0.0
*
       B21 =.035250/(JDB)
*
       B22 = -1.0/(JDB)
*
       B31 = 1.0/TC
*
       B32 = 0.0
*
*
DERIVATIVE
NOSORT
*
*    COMPUTE INPUT TO THE NONLINEAR MODEL, MF(T), WW(T).
*    RUN #9
          DMF =(MFDLY - MFI)/MFB
          DQD =(QDD - QDI)/QDB
          DNGDOT  = A11*DNG + A12*DNS + A13*DE + B11*DMF
          DNSDOT  = A21*DNG + A22*DNS + A23*DE + B21*DMF + B22*DQD
          DEDOT   = A33*DE  + B31*DMF
*
*
*   DYNAMIC EQUATIONS FOR LINEAR MODEL.
          DNG=INTGRL(0.0,DNGDOT)
          DNS=INTGRL(0.0,DNSDOT)
          DE =INTGRL(0.0,DEDOT)
          NGL = NGI + DNG*NGB
          NSL = NSI + DNS*NSB
SORT
```

```
*
*   THE STATEMENTS IN THE PREVIOUS (DERIVATIVE) SECTION YIELD VALUES
*   OF 'NG', AND 'NS' AS CALCULATED BY THE NONLINEAR AND STATE SPACE
*   MODELS.  THE STATEMENTS BELOW COMPUTE THE VALUES OF 'NG', AND 'NS'
*   AS RECORDED FROM GAS TURBINE TEST DATA.
*
TERMINAL
METHOD RKS
CONTROL FINTIM=50.0,DELT=0.001
PRINT 0.5,NGD,NGL,NSD,NSL,MFD,QDD
END
STOP
```

```
**********************************************************************
*                                                                    *
*                                                                    *
*              BOEING MODEL 502-6A GAS TURBINE                       *
*                                                                    *
*               DYNAMIC COMPUTER SIMULATION                          *
*                                                                    *
*                      MODIFIED BY                                   *
*                       S.D. METZ                                    *
*                   FROM  ROUTINES BY                                *
*              V.J.HERDA AND V.A. STAMMETTI                          *
*                                                                    *
*     THIS PROGRAM SIMULATES THE DYNAMIC RESPONSE OF THE NPS        *
*   BOEING GAS TURBINE TEST FACILITY USING A MULTILPLE              *
*   LINEARIZATION TECHNIQUE.                                        *
*                                                                    *
**********************************************************************
C
C
PARAM JG=0.009525, JD=0.3000, PI=3.14159, TC=1.00
*
*   THE FOLLOWING VALUES LISTED ON THE FUNCTION CARD ARE FOR FUEL FLOW,
*   GAS GENERATOR SPEED, TORQUE AND DYNO SPEED AS A FUNCTION OF TIME.
*   THESE VALUES WERE OBTAINED FROM STRIP CHART RECORDS AND ARE ENTERED
*   IN THE FORM (E.G. FUEL FLOW)  ...TIME(SEC), FUEL FLOW.....
*
*
C
C        THIS SET IS FOR EXPERIMENTAL RUN # 3.
C
C
AFGEN NGDATA= 0.0,20220.0, 3.55,20220.0, 3.7,20315.7, 3.8,20411.4, ...
              4.0,20698.4, 5.0,23186.1, 6.0,25482.4, 7.0,27204.6, ...
            8.0,28926.9, 9.0,30649.1, 10.0,32180.0, 10.5,32371.4, ...
           11.0,32180.0, 11.5,32084.3, 12.0,32180.0, 50.0,32180.0
AFGEN NSDATA= 0.0,506.0, 2.0,505.7, 3.6,505.5, 4.0,506.5, ...
         5.0,508.3, 6.0,509.7, 7.0,511.1, 8.0,512.5, 9.0,513.5, ...
         10.0,514.4, 10.4,514.7, 11.0,514.4, 12.0,515.1, ...
         13.0,515.8, 14.0,516.7, 16.3,517.2, 17.0,516.7, ...
         18.0,515.8, 19.0,514.9, 20.0,513.0, 50.0,513.0
AFGEN MFDATA= 0.0,77.5, 3.0,77.5, 3.3,79.6, 3.6,85.8, ...
         4.0,91.99, 5.0,106.5, 6.0,118.9, 7.0,130.5, 8.0,141.3, ...
         9.0,156.2, 10.0,172.7, 11.0,156.2, 12.0,160.3, 50.0,160.3


AFGEN QDDATA= 0.0,125.0, 3.6,125.0, 3.8,128.4, 4.0,131.8, ...
         5.0,172.4, 6.0,226.7, 7.0,274.1, 8.0,321.6, 9.0,375.8, ...
         10.0,430.0, 10.5,443.6, 11.0,430.0, 12.0,416.4, ...
         13.0,416.4, 14.0,419.8, 15.0,416.4, 18.0,423.0, ...
         20.0,430.0, 50.0,430
*
*
INITIAL
*   ESTABLISH INITIAL CONDITIONS.
*
        NGI=20220.0
        NSI=506.0
        MFI = 77.5
        QDI = 125.0
*
*
*
*  SET INITIAL STATE PERTURBATION TO ZERO
*
        DNG = 0.0
        DNS = 0.0
        DE  = 0.0
```

```
          NGB = 26000.0
          NSB = 1500.0
          MFB = 122.5
          QDB = 220.0
          QCB = 80.0
          JGB = (NGB/QCB)*JG
          JDB = (NSB/QDB)*JD
DYNAMIC
*
*         DATA CURVE FORMULATION
*
          NGD = AFGEN(NGDATA,TIME)
          NSD = AFGEN(NSDATA,TIME)
          MFD = AFGEN(MFDATA,TIME)
          QDD = AFGEN(QDDATA,TIME)
        MFDLY = TRANSP(100,MFI,0.70,MFD)
*
*        STATE SPACE LINEAR MODEL FORMULATION
*
      A11 = (20.282-32.560*(NGL/NGB))/(JGB)
*
      A12 = (-0.448)/(JGB)
*
      A13 = (13.624)/(JGB)
*
      A21 = (2.1131+.91786*(NGL/NGB))/(JDB)
*
      A22 = (-.0998-0.445*(NGL/NGB))/(JDB)
*
      A23 =(-.18851)/(JDB)
*
      A31 = 0.0
*
      A32 = 0.0
*
      A33 = -1.0/TC
*
      B11 = 0.0
*
      B12 = 0.0
*
      B21 =.035250/(JDB)
*
      B22 = -1.0/(JDB)
*
      B31 = 1.0/TC
*
      B32 = 0.0
*
*
DERIVATIVE
NOSORT
*
*   COMPUTE INPUT TO THE NONLINEAR MODEL, MF(T), WW(T).
*   RUN #3
         DMF =(MFDLY - MFI)/MFB
         DQD =(QDD - QDI)/QDB
         DNGDOT  = A11*DNG + A12*DNS + A13*DE + B11*DMF
         DNSDOT  = A21*DNG + A22*DNS + A23*DE + B21*DMF + B22*DQD
         DEDOT   = A33*DE  + B31*DMF
*
*
*   DYNAMIC EQUATIONS FOR LINEAR MODEL.
         DNG=INTGRL(0.0,DNGDOT)
         DNS=INTGRL(0.0,DNSDOT)
         DE =INTGRL(0.0,DEDOT)
         NGL = NGI + DNG*NGB
         NSL = NSI + DNS*NSB
SORT
```

```
*   THE STATEMENTS IN THE PREVIOUS (DERIVATIVE) SECTION YIELD VALUES
*   OF 'NG', AND 'NS' AS CALCULATED BY THE NONLINEAR AND STATE SPACE
*   MODELS.  THE STATEMENTS BELOW COMPUTE THE VALUES OF 'NG', AND 'NS'
*   AS RECORDED FROM GAS TURBINE TEST DATA.
*
TERMINAL
METHOD RK5
CONTROL FINTIM=50.0,DELT=0.001
PRINT 0.50,NGD,NGL,NSD,NSL,MFD,QDD,DNSDOT,A21,DNG, ...
       A22,DNS,A23,DE,B21,DMF,B22,DQD
END
STOP
```

```
**********************************************************************
*                                                                    *
*                                                                    *
*                  BOEING MODEL 502-6A GAS TURBINE                   *
*                                                                    *
*                    DYNAMIC COMPUTER SIMULATION                     *
*                                                                    *
*                           MODIFIED BY                              *
*                            S.D. METZ                               *
*                         FROM  ROUTINES BY                          *
*                   V.J.HERDA AND V.A. STAMMETTI                     *
*                                                                    *
*       THIS PROGRAM SIMULATES THE DYNAMIC RESPONSE OF THE NPS       *
*    BOEING GAS TURBINE TEST FACILITY USING A MULTILPLE              *
*    LINEARIZATION TECHNIQUE.                                        *
*                                                                    *
**********************************************************************
C
C
PARAM JG=0.009525, JD=0.3000, PI=3.14159, TC=1.00
*
*   THE FOLLOWING VALUES LISTED ON THE FUNCTION CARD ARE FOR FUEL FLOW,
*   GAS GENERATOR SPEED, TORQUE AND DYNO SPEED AS A FUNCTION OF TIME.
*   THESE VALUES WERE OBTAINED FROM STRIP CHART RECORDS AND ARE ENTERED
*   IN THE FORM (E.G. FUEL FLOW)  ...TIME(SEC), FUEL FLOW.....
*
*
C
C        THIS SET IS FOR EXPERIMENTAL RUN # 5.
C
C
AFGEN NGDATA= 0.0,20220.0, 3.1,20220.0, 3.6,20412.1, 4.0,21372.6, ...
         5.0,23677.7, 6.0,25886.9, 7.0,27711.8, 8.0,29440.6, ...
         9.0,30977.4, 10.0,32206.8, 10.2,32322.1, 10.8,32130.0, ...
        11.5,32130.0, 12.1,32014.7, 13.4,32206.8, 14.5,32014.7, ...
        15.6,32206.8, 16.8,32014.7, 17.0,32206.8, 50.0,32130.0
*
AFGEN NSDATA= 0.0,1491.6, 1.0,1490.4, 3.0,1489.6, 4.0,1492.5, ...
         5.0,1501.0, 6.0,1508.3, 7.0, 1511.7, 7.8,1513.3, ...
         9.6,1513.3, 10.1,1513.8, 12.0,1508.3, 13.0,1510.0, ...
         50.0,1510.0
*
AFGEN MFDATA= 0.0,83.75, 3.2,83.75, 5.0,109.6, 6.0,122.2, ...
         7.0,135.5, 8.0,150.2, 9.0,162.8, 9.8,172.4, ...
         10.0,168.7, 11.0,165.0, 50.0,165.0
*
AFGEN QDDATA= 0.0,71.0, 4.4,71.0, 5.0,78.7, 6.0,94.1, ...
         7.0,107.5, 8.0,120.9, 9.0,134.4, 10.2,149.8, ...
         10.9,147.9, 11.6,147.9, 12.4,143.2, 13.0,145.9, ...
         13.5,144.0, 14.0,145.9, 15.0,144.0, 50.0,144.0
*
*
*
INITIAL
*   ESTABLISH INITIAL CONDITIONS.
*
        NGI=20220.0
        NSI=1491.6
        MFI = 83.75
        QDI = 71.0
*
*
*
*  SET INITIAL STATE PERTURBATION TO ZERO
*
        DNG = 0.0
        DNS = 0.0
        DE  = 0.0
```

```
          NGB = 26000.0
          NSB = 1500.0
          MFB = 122.5
          QDB = 220.0
          QCB = 80.0
          JGB = (NGB/QCB)*JG
          JDB = (NSB/QDB)*JD
DYNAMIC
*
*         DATA CURVE FORMULATION
*
          NGD = AFGEN(NGDATA,TIME)
          NSD = AFGEN(NSDATA,TIME)
          MFD = AFGEN(MFDATA,TIME)
          QDD = AFGEN(QDDATA,TIME)
        MFDLY = TRANSP(100,MFI,0.70,MFD)
*
*        STATE SPACE LINEAR MODEL FORMULATION
*
      A11 = (20.282-32.560*(NGL/NGB))/(JGB)
*
      A12 = (-0.4480)/(JGB)
*
      A13 = (13.624)/(JGB)
*
      A21 = (2.1131+.91786*(NGL/NGB))/(JDB)
*
      A22 = (-.0998-0.445*(NGL/NGB))/(JDB)
*
      A23 =(-0.18851)/(JDB)
*
      A31 = 0.0
*
      A32 = 0.0
*
      A33 = -1.0/TC
*
      B11 = 0.0
*
      B12 = 0.0
*
      B21 =.035250/(JDB)
*
      B22 = -1.0/(JDB)
*
      B31 = 1.0/TC
*
      B32 = 0.0
*
*
DERIVATIVE
NOSORT
*
*    COMPUTE INPUT TO THE NONLINEAR MODEL, MF(T), WW(T).
*    RUN #9
          DMF =(MFDLY - MFI)/MFB
          DQD =(QDD - QDI)/QDB
          DNGDOT  = A11*DNG + A12*DNS + A13*DE + B11*DMF
          DNSDOT  = A21*DNG + A22*DNS + A23*DE + B21*DMF + B22*DQD
          DEDOT   = A33*DE  + B31*DMF
*
*
*   DYNAMIC EQUATIONS FOR LINEAR MODEL.
          DNG=INTGRL(0.0,DNGDOT)
          DNS=INTGRL(0.0,DNSDOT)
          DE =INTGRL(0.0,DEDOT)
          NGL = NGI + DNG*NGB
          NSL = NSI + DNS*NSB
SORT
```

```
*   THE STATEMENTS IN THE PREVIOUS (DERIVATIVE) SECTION YIELD VALUES
*   OF 'NG', AND 'NS' AS CALCULATED BY THE NONLINEAR AND STATE SPACE
*   MODELS.  THE STATEMENTS BELOW COMPUTE THE VALUES OF 'NG', AND 'NS'
*   AS RECORDED FROM GAS TURBINE TEST DATA.
*
TERMINAL
METHOD RK5
CONTROL FINTIM=50.0,DELT=0.001
PRINT 0.5,NGD,NGL,NSD,NSL,MFD,QDD,DNSDOT,A21,DNG,A22,DNS, ...
      A23,DE,B21,DMF,B22,DQD
END
STOP
```

```
**********************************************************************
*                                                                    *
*                                                                    *
*                BOEING MODEL 502-6A GAS TURBINE                     *
*                                                                    *
*                  DYNAMIC COMPUTER SIMULATION                       *
*                                                                    *
*                          MODIFIED BY                               *
*                           S.D. METZ                                *
*                       FROM  ROUTINES BY                            *
*                V.J.HERDA AND V.A. STAMMETTI                        *
*                                                                    *
*      THIS PROGRAM SIMULATES THE DYNAMIC RESPONSE OF THE NPS        *
*   BOEING GAS TURBINE TEST FACILITY USING A MULTILPLE               *
*   LINEARIZATION TECHNIQUE.                                         *
*                                                                    *
**********************************************************************
C
C
PARAM JG=0.009525, JD=0.3000, PI=3.14159, TC=1.00
*
*   THE FOLLOWING VALUES LISTED ON THE FUNCTION CARD ARE FOR FUEL FLOW,
*   GAS GENERATOR SPEED, TORQUE AND DYNO SPEED AS A FUNCTION OF TIME.
*   THESE VALUES WERE OBTAINED FROM STRIP CHART RECORDS AND ARE ENTERED
*   IN THE FORM (E.G. FUEL FLOW)  ...TIME(SEC), FUEL FLOW.....
*
*
C
C        THIS SET IS FOR EXPERIMENTAL RUN # 7.
C
C
AFGEN NGDATA= 0.0,20220.0, 4.0,20220.0, 4.5,20759.7, 5.0,21954.7, ...
              6.0,24460.3, 7.0,26541.9, 8.0,28122.4, 9.0,29780.0, ...
           10.0,31399.0, 10.4,32092.9, 10.6,32362.7, 11.0,32285.6, ...
           12.0,32170.0, 15.0,32170.0, 20.0,32170
AFGEN NSDATA= 0.0,2252.0, 1.0,2245.4, 2.0,2238.8, 3.0,2232.2, ...
         4.0,2225.6, 5.0,2278.4, 6.0,2436.8, 7.0,2568.8, ...
         7.7,2621.6, 8.0,2611.0, 9.0,2579.4, 10.0,2568.8, ...
         11.0,2542.4, 13.0,2516.0, 15.0,2516.0, 20.0,2516
AFGEN MFDATA= 0.0,78.8, 1.0,78.8, 3.0,78.8, 4.0,87.1, ...
         4.5,95.3, 5.0,103.6, 6.0,111.9, 7.0,128.5, 8.0,136.8, ...
         9.0,161.7, 10.0,174.1, 10.4,178.3, 11.0,170.0, ...
         15.0,170.0, 20.0,170.0

AFGEN QDDATA= 0.0,25.0, 2.0,25.0, 4.0,25.0, 5.0,25.0, ...
         6.0,49.6, 7.0,92.6, 8.0,129.4, 9.0,178.6, ...
         10.0,233.9, 11.0,264.6, 12.0,252.3, 13.0,246.1, ...
         15.0,240.0, 20.0,240.0
*
*
*
INITIAL
*
*   ESTABLISH INITIAL CONDITIONS.
        NGI=20220.0
        NSI=2252.0
        MFI = 78.75
        QDI = 25.0
*
*
*
*  SET INITIAL STATE PERTURBATION TO ZERO
*
        DNG = 0.0
        DNS = 0.0
        DE  = 0.0
```

```
          NGB = 26000.0
          NSB = 1500.0
          MFB = 122.5
          QDB = 220.0
          QCB = 80.0
          JGB = (NGB/QCB)*JG
          JDB = (NSB/QDB)*JD
DYNAMIC
*
*         DATA CURVE FORMULATION
*
          NGD = AFGEN(NGDATA,TIME)
          NSD = AFGEN(NSDATA,TIME)
          MFD = AFGEN(MFDATA,TIME)
          QDD = AFGEN(QDDATA,TIME)
        MFDLY = TRANSP(100,MFI,0.70,MFD)
*
*        STATE SPACE LINEAR MODEL FORMULATION
*
      A11 = (20.282-32.560*(NGL/NGB))/(JGB)
*
      A12 = (-0.448)/(JGB)
*
      A13 = (13.624)/(JGB)
*
      A21 = (2.1131+.91786*(NGL/NGB))/(JDB)
*
      A22 = (-.0998-0.445*(NGL/NGB))/(JDB)
*
      A23 = (-.18851)/(JDB)
*
      A31 = 0.0
*
      A32 = 0.0
*
      A33 = -1.0/TC
*
      B11 = 0.0
*
      B12 = 0.0
*
      B21 =.035250/(JDB)
*
      B22 = -1.0/(JDB)
*
      B31 = 1.0/TC
*
      B32 = 0.0
*
*
DERIVATIVE
NOSORT
*
*    COMPUTE INPUT TO THE NONLINEAR MODEL, MF(T), WW(T).
*    RUN #7
*
           DMF =(MFDLY - MFI)/MFB
          DQD =(QDD - QDI)/QDB
          DNGDOT   = A11*DNG + A12*DNS + A13*DE + B11*DMF
          DNSDOT   = A21*DNG + A22*DNS + A23*DE + B21*DMF + B22*DQD
          DEDOT    = A33*DE  + B31*DMF
*
*
*   DYNAMIC EQUATIONS FOR LINEAR MODEL.
          DNG=INTGRL(0.0,DNGDOT)
          DNS=INTGRL(0.0,DNSDOT)
          DE =INTGRL(0.0,DEDOT)
          NGL = NGI + DNG*NGB
          NSL = NSI + DNS*NSB
SORT
*
```

```
*   OF 'NG', AND 'NS' AS CALCULATED BY THE NONLINEAR AND STATE SPACE
*   MODELS.  THE STATEMENTS BELOW COMPUTE THE VALUES OF 'NG', AND 'NS'
*   AS RECORDED FROM GAS TURBINE TEST DATA.
*
TERMINAL
METHOD RK5
CONTROL FINTIM=20.0,DELT=0.001
PRINT 0.25,NGD,NGL,NSD,NSL,MFD,QDD
END
STOP
```

```
*********************************************************************
*                                                                   *
*                                                                   *
*               BOEING MODEL 502-6A GAS TURBINE                     *
*                                                                   *
*                 DYNAMIC COMPUTER SIMULATION                       *
*                                                                   *
*                         MODIFIED BY                               *
*                          S.D. METZ                                *
*                       FROM  ROUTINES BY                           *
*                V.J.HERDA AND V.A. STAMMETTI                       *
*                                                                   *
*     THIS PROGRAM SIMULATES THE DYNAMIC RESPONSE OF THE NPS        *
*   BOEING GAS TURBINE TEST FACILITY USING A MULTILPLE              *
*   LINEARIZATION TECHNIQUE.                                        *
*                                                                   *
*********************************************************************
C
C
PARAM JG=0.009525, JD=0.3000, PI=3.14159, TC=1.00
*
*   THE FOLLOWING VALUES LISTED ON THE FUNCTION CARD ARE FOR FUEL FLOW,
*   GAS GENERATOR SPEED, TORQUE AND DYNO SPEED AS A FUNCTION OF TIME.
*   THESE VALUES WERE OBTAINED FROM STRIP CHART RECORDS AND ARE ENTERED
*   IN THE FORM (E.G. FUEL FLOW)  ...TIME(SEC), FUEL FLOW.....
*
*
C
C        THIS SET IS FOR EXPERIMENTAL RUN # 9.
C
C
AFGEN NGDATA= 0.0,32150.0, 5.0,32150.0, 10.0,32150.0, 15.0,32150.0, ...
         20.0,32150.0, 25.0,32150.0, 50.0,32150.0
AFGEN NSDATA= 0.0,510.0, 2.0,510.0, 3.0,576.1, 4.0,642.14, ...
         5.0,708.2, 6.0,774.3, 7.0,840.4, 8.0,922.9, 9.0,1005.5, ...
         10.0,1137.7, 11.0,1269.8, 12.0,1435.0, 13.0,1583.6, ...
         14.0,1798.4, 15.0,1963.5, 16.0,2128.7, 17.0,2227.8, ...
         18.0,2326.9, 19.0,2369.0, 20.0,2409.5, 21.0,2459.1, ...
         22.0,2492.1, 23.0,2508.6, 24.0,2525.0, 25.0,2525.0, ...
         50.0,2525.0
AFGEN MFDATA= 0.0,155.8, 3.0,155.8, 5.0,155.8, 7.0,160.0, ...
         10.0,160.0, 13.0,164.1, 15.0,164.1, 18.0,164.1, ...
         20.0,168.3, 23.0,168.3, 24.0,168.3, 25.0,168.3, 50.0,168.3


AFGEN QDDATA= 0.0,394.0, 2.0,394.0, 3.0,382.5, 4.0,371.0, ...
         5.0,359.5, 7.0,336.5, 8.0,325.0, 9.0,313.50, ...
         10.0,302.0, 11.0,284.8, 12.0,261.8, 13.0,250.3, ...
         14.0,233.0, 15.0,221.5, 16.0,221.5, 19.0,221.5, ...
         20.0,221.5, 21.0,210.0, 23.0,210.0, 25.0,210.0, 50.0,210.0
*
*
*
INITIAL
*
*   ESTABLISH INITIAL CONDITIONS.
*
        NGI=32150.0
        NSI=510.0
        MFI = 155.8
        QDI = 394.0
*
*
*
*  SET INITIAL STATE PERTURBATION TO ZERO
*
        DNG = 0.0
        DNS = 0.0
        DE  = 0.0
```

```
          NGB = 26000.0
          NSB = 1500.0
          MFB = 122.5
          QDB = 220.0
          QCB = 80.0
          JGB = (NGB/QCB)*JG
          JDB = (NSB/QDB)*JD
DYNAMIC
*
*         DATA CURVE FORMULATION
*
          NGD = AFGEN(NGDATA,TIME)
          NSD = AFGEN(NSDATA,TIME)
          MFD = AFGEN(MFDATA,TIME)
          QDD = AFGEN(QDDATA,TIME)
        MFDLY = TRANSP(100,MFI,0.70,MFD)
*
*         STATE SPACE LINEAR MODEL FORMULATION
*
      A11 = (20.282-32.560*(NGL/NGB))/(JGB)
*
      A12 = (-0.4480)/(JGB)
*
      A13 = (13.624)/(JGB)
*
      A21 = (2.1131+.91786*(NGL/NGB))/(JDB)
*
      A22 = (-.0998-0.445*(NGL/NGB))/(JDB)
*
      A23 =(-0.18851)/(JDB)
*
      A31 = 0.0
*
      A32 = 0.0
*
      A33 = -1.0/TC
*
      B11 = 0.0
*
      B12 = 0.0
*
      B21 =.035250/(JDB)
*
      B22 = -1.0/(JDB)
*
      B31 = 1.0/TC
*
      B32 = 0.0
*
*
DERIVATIVE
NOSORT
*
*   COMPUTE INPUT TO THE NONLINEAR MODEL, MF(T), WW(T).
*   RUN #9
         DMF =(MFDLY - MFI)/MFB
         DQD =(QDD - QDI)/QDB
         DNGDOT  = A11*DNG + A12*DNS + A13*DE + B11*DMF
         DNSDOT  = A21*DNG + A22*DNS + A23*DE + B21*DMF + B22*DQD
         DEDOT   = A33*DE  + B31*DMF
*
*
*   DYNAMIC EQUATIONS FOR LINEAR MODEL.
         DNG=INTGRL(0.0,DNGDOT)
         DNS=INTGRL(0.0,DNSDOT)
         DE =INTGRL(0.0,DEDOT)
         NGL = NGI + DNG*NGB
         NSL = NSI + DNS*NSB
SORT
```

```
*
*   THE STATEMENTS IN THE PREVIOUS (DERIVATIVE) SECTION YIELD VALUES
*   OF 'NG', AND 'NS' AS CALCULATED BY THE NONLINEAR AND STATE SPACE
*   MODELS.  THE STATEMENTS BELOW COMPUTE THE VALUES OF 'NG', AND 'NS'
*   AS RECORDED FROM GAS TURBINE TEST DATA.
*
TERMINAL
METHOD RK5
CONTROL FINTIM=50.0,DELT=0.001
PRINT 0.5,NGD,NGL,NSD,NSL,MFD,QDD,DNSDOT,A21,DNG, ...
          A22,DNS,A23,DE,B21,DMF,B22,DQD
END
STOP
```

APPENDIX E

SOURCE DATA RUNS:

Run 1: Data run for dynamic Ns run at constant Ng.

Ng -- 20230 - 20240 rpm	Ns -- 505 - 2215 rpm
Qd -- 125 - 25 lbft	Mf -- 78.4 - 82.1 lbm/hr

Run 3: Data run for dynamic Ng run at constant Ns.

Ng -- 20220 - 32180 rpm	Ns -- 506 - 513 rpm
Qd -- 125 - 430 lbft	Mf -- 77.5 - 160.3 lbm/hr

Run 5: Data run for dynamic Ng run at constant Ns

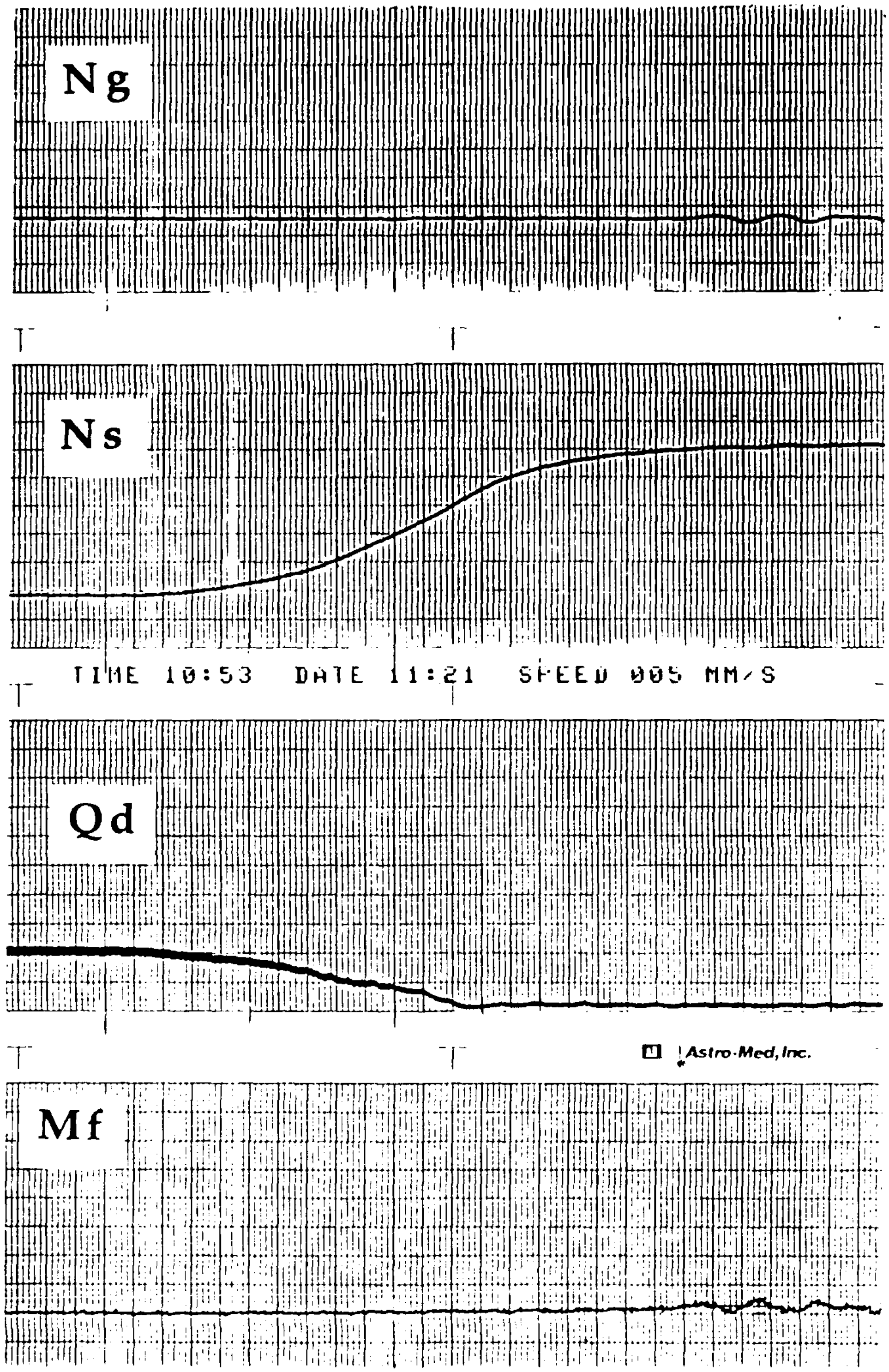
Ng -- 20220 - 32130 rpm	Ns -- 1492 - 1510 rpm
Qd -- 71 - 144 lbft	Mf -- 83.8 - 165 lbm/hr

Run 7 : Data run for dynamic Ng run at Ns (2252-2516 rpm).

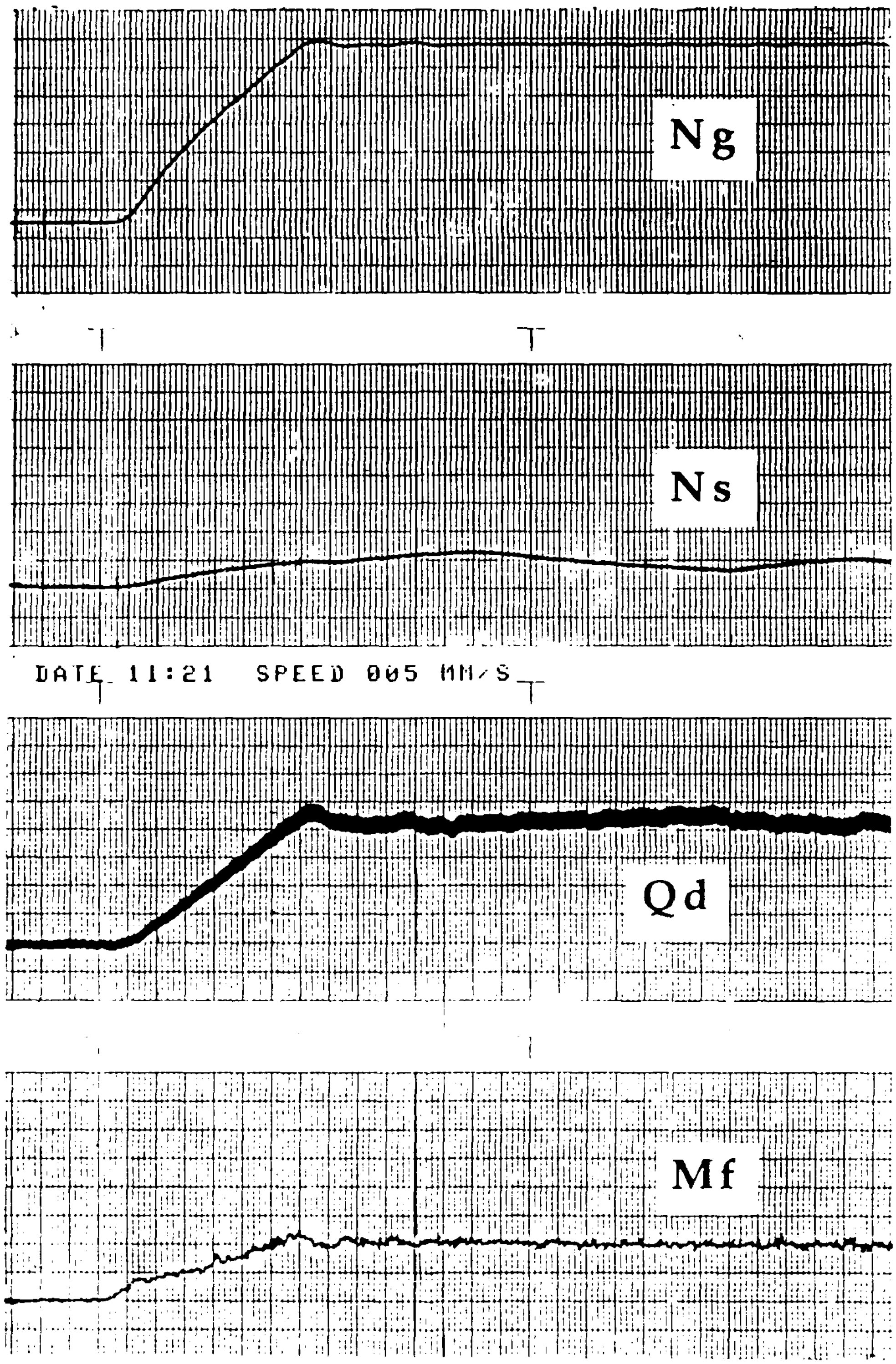
Ng -- 20220 - 32170 rpm	Ns -- 2252 - 2516 rpm
Qd -- 25 - 240 lbft	Mf -- 78.8 - 170 lbm/hr

Run 9: Data run for dynamic Ns run at constant Ng.

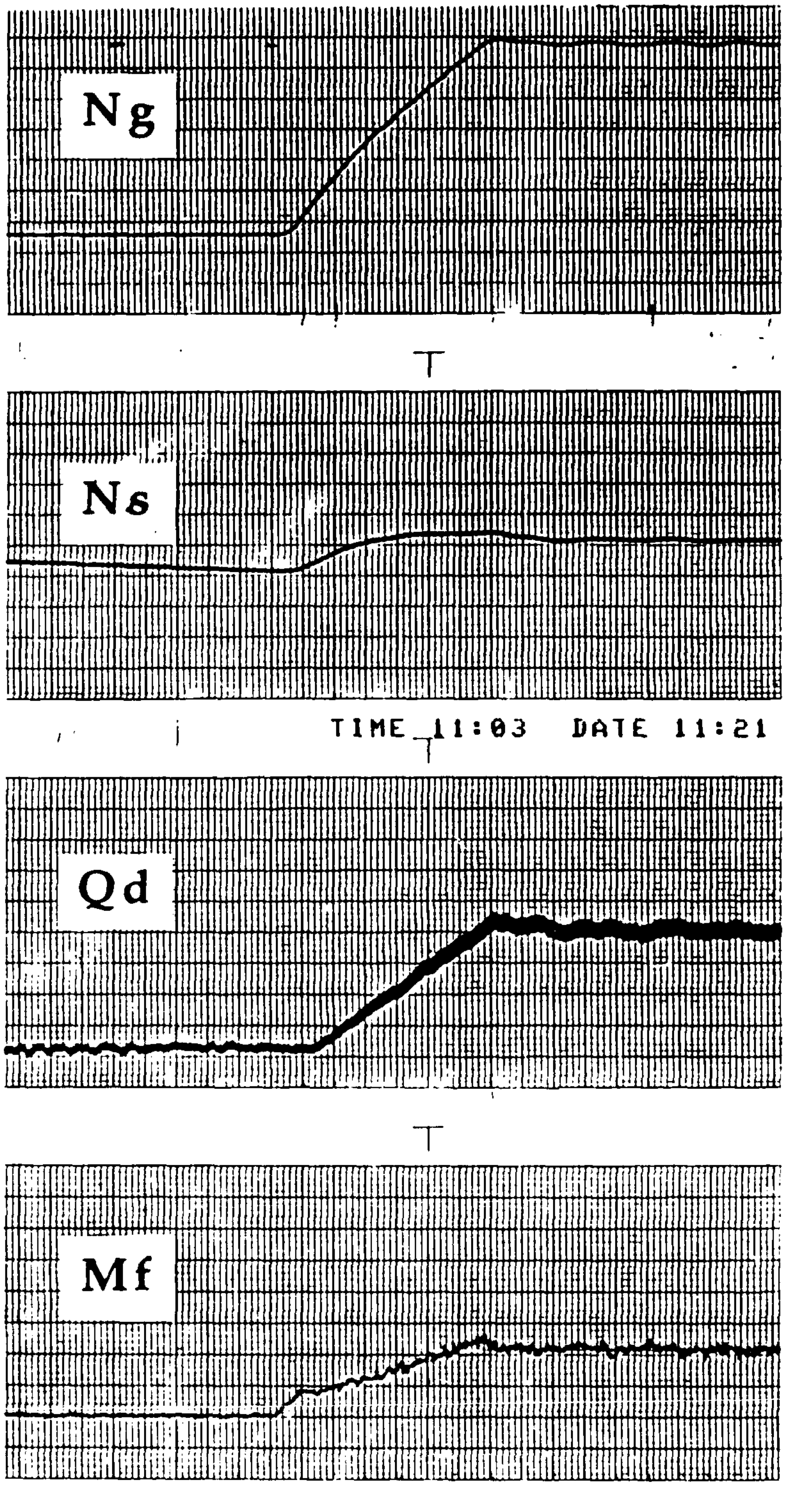
Ng -- 32150 rpm	Ns -- 510 - 2525 rpm
Qd -- 394 - 210 lbft	Mf -- 155.8 - 168.3 lbm/hr



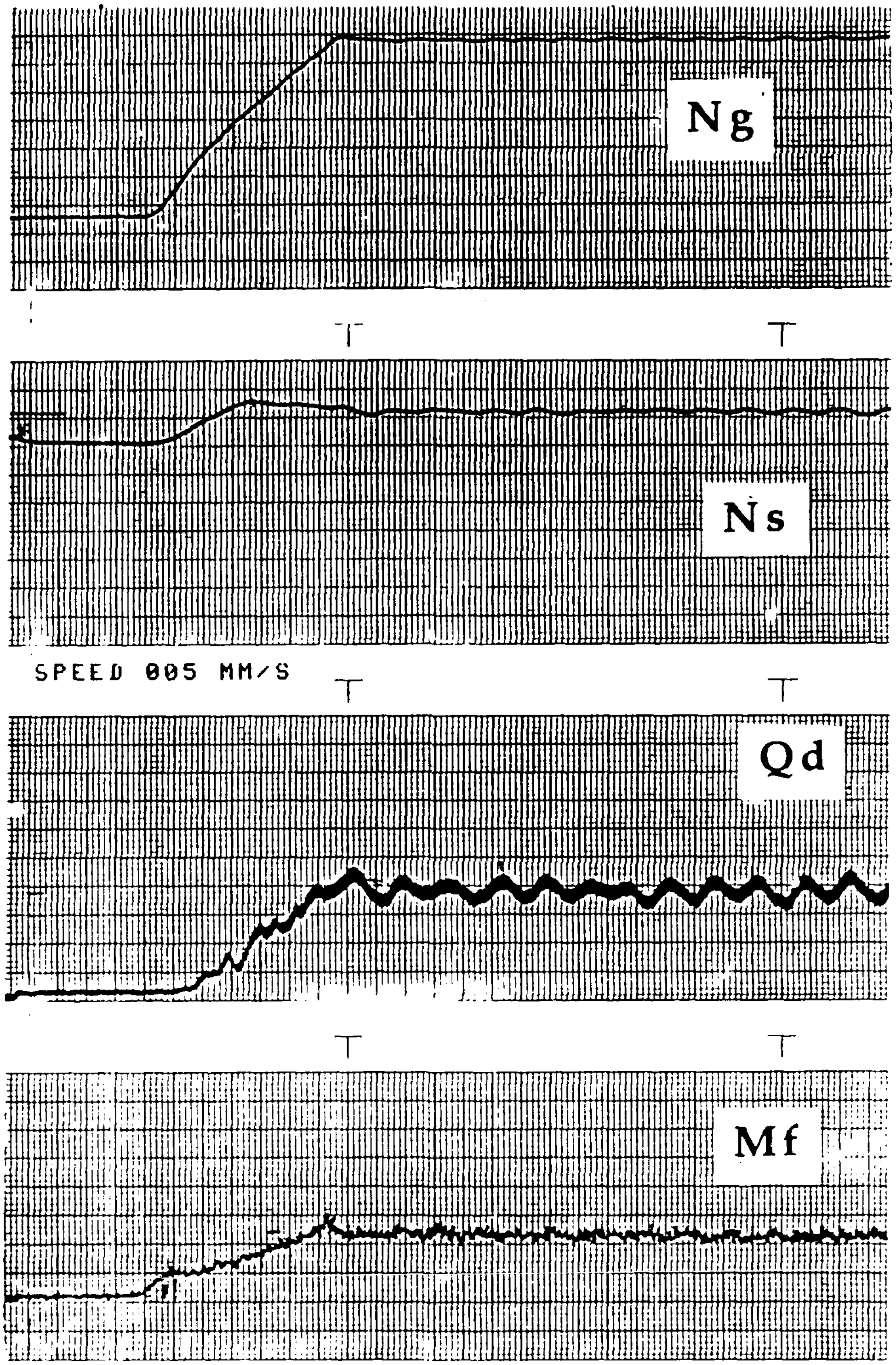
Run 1



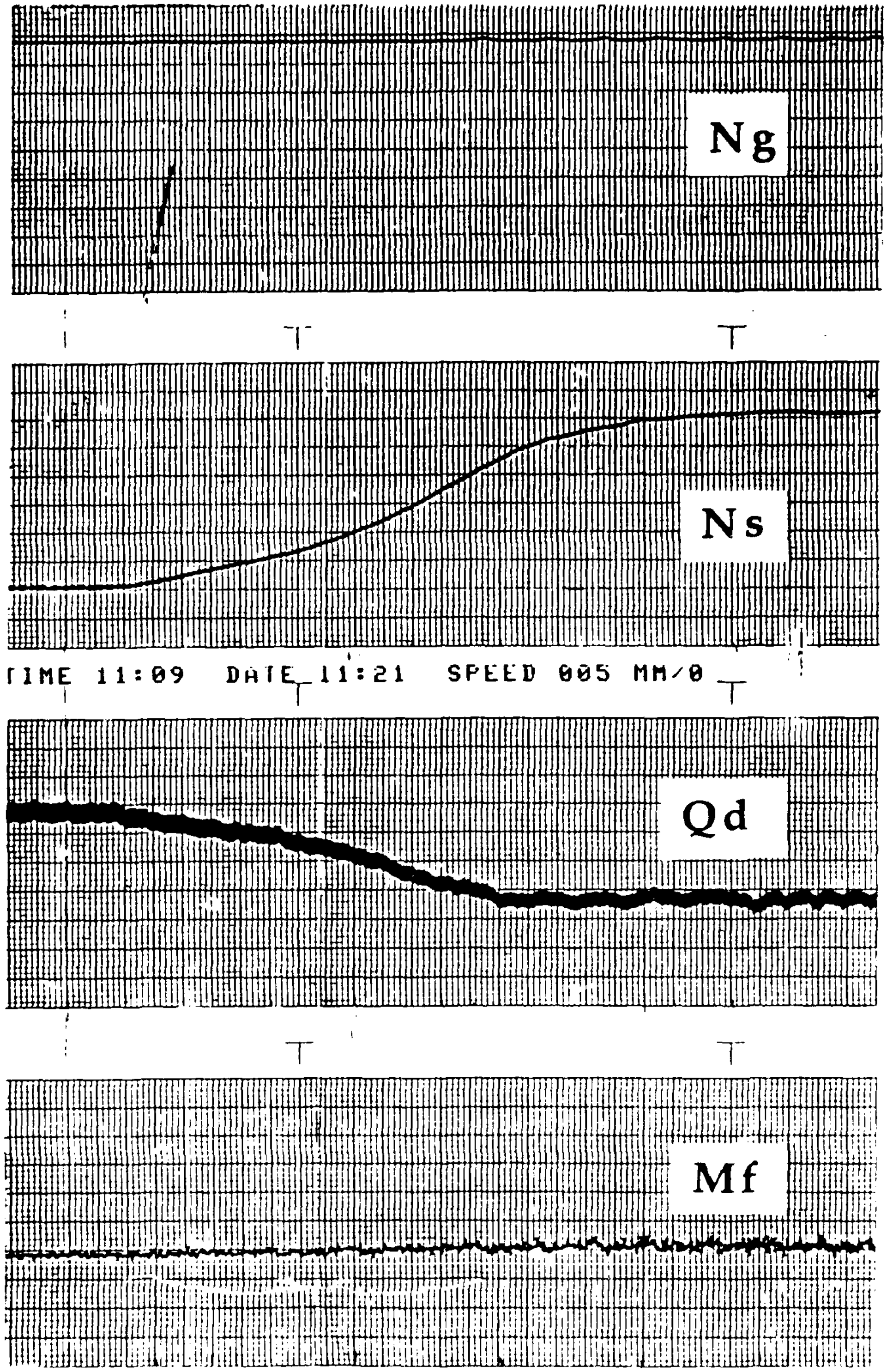
Run 3



Run 5



Run 7



Run 9

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